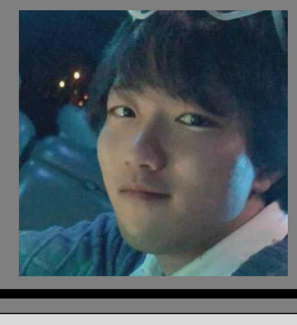
Spacetime Thermodynamics: to the Entropic Gravity

Koki Tokeshi (U.Tokyo/RESCEU, M1)



Abstract **Spacetime has thermodynamic aspects**: Einstein equation evaluated at a horizon represents the 1st law of thermodynamics. Inversely, one can derive the equation from thermodynamic considerations. Moreover, there is another derivation of Einstein equation: spacetime's entropy functional, required to be extremum with respect to the "displacement vector," yields the desired equation. This implies that $g_{\mu\nu}$ **is no longer a dynamical variable of gravitational theory**.

The fact that the entropy of a black hole is proportional to its surface area, not to its volume, implies that **all information is encoded in the surface**. Actually, Einstein-Hilbert Lagrangian density has intrinsically "**holographic structure**." The surface term of the full action corresponds to the entropy, while the bulk term does the all energy. Then the full gravitational action represents Helmholtz potential of spacetime.

Keywords Spacetime thermodynamics, Area law, Holographic principle, Entropic gravity

■ General Relativity ↔ Thermodynamics

1. Einstein equation at BH horizon → The 1st law

Setup: Consider a spacetime described by

$$\mathrm{d}s^2 = -f(r)(c\mathrm{d}t)^2 + \frac{\mathrm{d}r^2}{g(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$$

Horizon: $r = a$ where $g(a) = 0$ and $f'(a) = g'(a)$

Temperature: $k_\mathrm{B}T = \dfrac{\hbar c g'(a)}{4\pi}$

Einstein equation evaluated between $r = a$ and $r = a + \mathrm{d}a$:

$$T\,\mathrm{d}\left(k_\mathrm{B}\frac{c^3}{G\hbar}\frac{\mathcal{A}}{4}\right) - \frac{c^4}{2G}\mathrm{d}a = P\,\mathrm{d}\left(\frac{4}{3}\pi a^3\right)$$

This is the 1st law of thermodynamics: $T\mathrm{d}S - \mathrm{d}E = P\mathrm{d}V$

The entropy is given by

$$S = k_\mathrm{B}\frac{c^3}{G\hbar}\frac{\mathcal{A}}{4}\,, \quad \mathcal{A} = 4\pi a^2 \qquad \textbf{: Area law}$$

→ **All microscopic degrees of freedom are encoded in the surface!**

(Recall Boltzmann's principle: $S = k_\mathrm{B}\log W$)

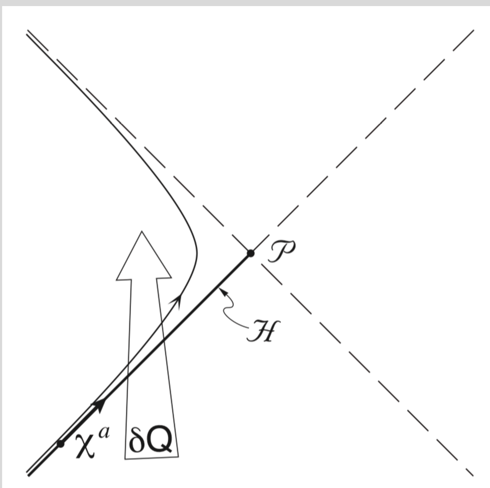
2. Thermodynamic considerations → Einstein equation

Assumptions:

$\delta Q = T\mathrm{d}S$: Clausius' relation,

$k_\mathrm{B}T = \dfrac{\hbar\kappa}{2\pi}$: Unruh temperature,

$S = \alpha\mathcal{A}$: Entropy proportionality.



L.H.S. of $\delta Q = T\mathrm{d}S$

$$\delta Q = \int_\mathcal{H}\mathrm{d}S^\mu J_\mu = -\kappa\int_\mathcal{H}\mathrm{d}\mathcal{A}\mathrm{d}\lambda\,\lambda T_{\mu\nu}k^\mu k^\nu$$

- $J_\mu = T_{\mu\nu}\xi^\nu$: conserved boost energy current of matter,
- $\mathrm{d}\mathcal{A}$: area element on a cross section of the horizon,
- λ : affine parameter of a geodesic,
- k^μ : a tangent vector to the horizon defined by $\xi^\mu = -\kappa\lambda k^\mu$

R.H.S. of $\delta Q = T\mathrm{d}S$

$$\mathrm{d}S = \alpha\,\mathrm{d}\mathcal{A} = \alpha\int_\mathcal{H}\mathrm{d}\mathcal{A}\mathrm{d}\lambda\,\theta = -\alpha\int_\mathcal{H}\mathrm{d}\mathcal{A}\mathrm{d}\lambda\,\lambda R_{\mu\nu}k^\mu k^\nu$$

- θ : expansion, defined by
  $$\theta \equiv \frac{1}{\mathrm{d}\mathcal{A}}\frac{\mathrm{d}}{\mathrm{d}\lambda}(\mathrm{d}\mathcal{A}) \quad\to\quad \mathrm{d}\mathcal{A} = \int_\mathcal{H}\mathrm{d}\mathcal{A}\mathrm{d}\lambda\,\theta$$
  $$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = -R_{\mu\nu}k^\mu k^\nu$$: Raychaudhuri eq. in thermal equilibrium

→ Einstein equation including the cosmological constant!

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{2\pi k_\mathrm{B}}{\hbar\alpha}T_{\mu\nu}$$

Derived Einstein equation from thermodynamic considerations

$\alpha = \dfrac{k_\mathrm{B}}{4G\hbar}$: consistent with Bekenstein-Hawking entropy

Λ : cosmological constant as an integration constant

→ Einstein equation may be regarded as a kind of **equation of state**. **Gravity is not a fundamental force**, **and it may not be correct to be canonically quantized**. This viewpoint leads to the idea of **entropic gravity**.

■ Holographic Structure of $\mathcal{L}_\mathrm{EH}$

The gravitational action in vacuum is given by

$$S_\mathrm{EH} = \frac{1}{16\pi G}\int\mathrm{d}^4x\,\mathcal{L}_\mathrm{EH} = \frac{1}{16\pi G}\int\mathrm{d}^4x\sqrt{-g}R$$

and $\mathcal{L}_\mathrm{EH}$ can be decomposed as $\mathcal{L}_\mathrm{EH} = \mathcal{L}_\mathrm{B} + \mathcal{L}_\mathrm{S}$ where

$$\mathcal{L}_\mathrm{B} = \sqrt{-g}g^{\mu\nu}({\Gamma^\lambda}_{\mu\rho}{\Gamma^\rho}_{\nu\lambda} - {\Gamma^\lambda}_{\mu\nu}{\Gamma^\rho}_{\rho\lambda})$$
$$\mathcal{L}_\mathrm{S} = \partial_\lambda[\sqrt{-g}(g^{\mu\nu}{\Gamma^\lambda}_{\mu\nu} - g^{\mu\lambda}{\Gamma^\nu}_{\mu\nu})]$$

These "**bulk**" and "**surface**" Lagrangians are related by

$$\sqrt{-g}\mathcal{L}_\mathrm{S} = -\partial_\lambda\left[g_{\mu\nu}\frac{\delta\mathcal{L}_\mathrm{B}}{\delta(\partial_\lambda g_{\mu\nu})}\right] \qquad \textbf{: Holographic relation}$$

→ **One can obtain the full action only from the information on the surface!**

※ The full action gives Helmholtz potential of the spacetime:

$$S_\mathrm{EH} = S_\mathrm{B} + S_\mathrm{S} = \frac{\mathcal{A}}{4} - \beta E = -\beta F$$

■ Theory of Entropic Gravity

Assumption: Gravity is an emergent phenomenon as a result of extremized spacetime entropy. Given that

$$S = -\int_\mathcal{V}\mathrm{d}^4x\sqrt{-g}\,(4{P_{\mu\nu}}^{\rho\sigma}\nabla_\rho\xi^\mu\nabla_\sigma\xi^\nu - T_{\mu\nu}\xi^\mu\xi^\nu)$$

ξ^μ : "deformation" of spacetime

$\delta S = 0$: varying **with respect to** ξ^μ

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \text{: Einstein equation again}$$

→ $g_{\mu\nu}$ **is no longer a fundamental dynamical variable** in a thermodynamic interpretation of gravity. **Gravity is an entropic force** **as a consequence of extremized entropy: microscopic degrees of freedom extremize entropy and consequently yield gravity**. ("**entropic gravity**" or "**emergent gravity**")

A simple example: Newtonian gravity as an emergent force

$k_\mathrm{B}T = \dfrac{\hbar|a|}{2\pi c}$: Unruh temperature,

$E = Mc^2$: energy of the source,

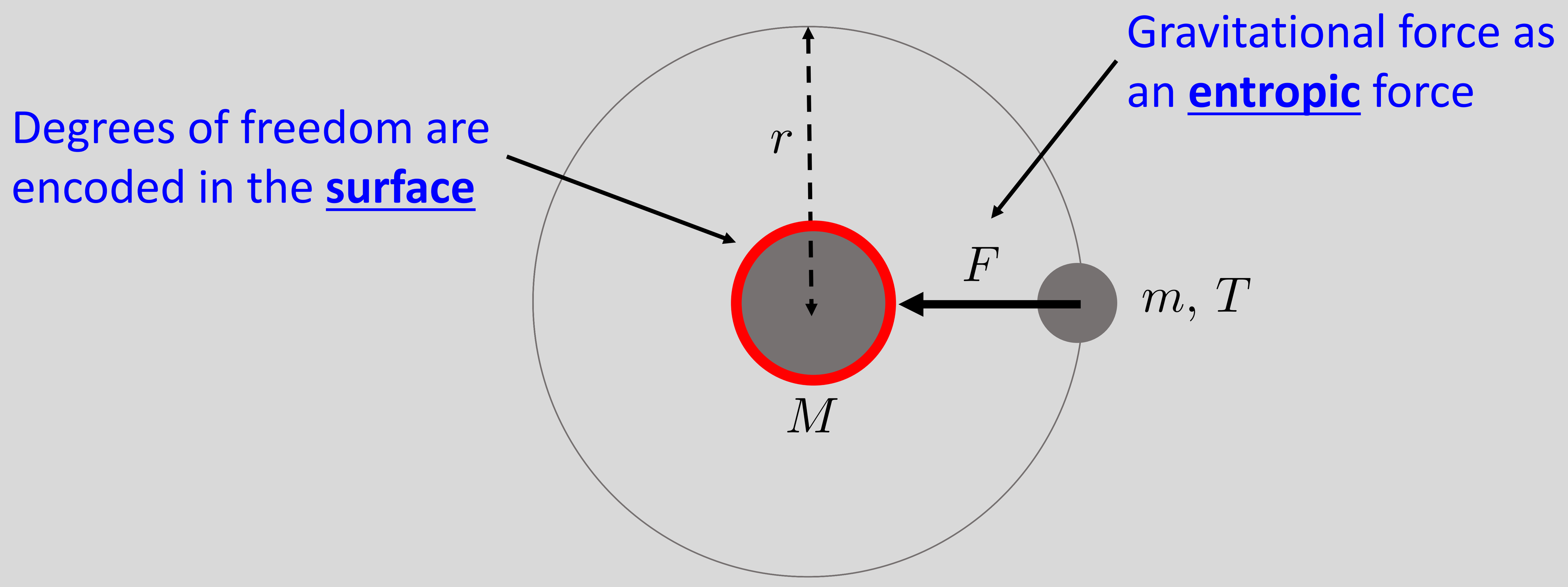
$E = \dfrac{N}{2}k_\mathrm{B}T$: energy encoded in the surface, for mass m,

$N = \dfrac{c^3}{G\hbar}\mathcal{A} = \dfrac{c^3}{G\hbar}4\pi r^2$: degrees of freedom encoded in the surface (**Holographic principle**)

Combining these equations and assumptions yields

$$ma = -G\frac{Mm}{r^2}$$: Newtonian law as an emergent force!

→ **Newtonian force is no other than an entropic force, yielded by thermal bath** T**, and we perceive this "emergent" force as gravity.**



References [1] T. Padmanabhan, *Thermodynamical aspects of gravity: new insights*, Rep. Prog. Phys. 73 (2010) 046901

[2] D. Moustos, *Gravity as a thermodynamic phenomenon*, arXiv:gr-qc/1701.08967 (2017)

[3] E. Verlinde, *On the origin of gravity and the laws of Newton*, JHEP 04 (2011) 029

Summer School 2019 @ Aichi 7/30 – 8/2