Quantum Fisher information for a massive scalar field in an expanding universe.

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Abstract

In an expanding universe, an initial vacuum quantum state evolves into entangled particle states. We consider the entanglement entropy and the quantum Fisher information associated to a massive scalar field in the asymptotic future of an expanding universe with scale factor $-1/(H\eta)$, in terms of both the expansion parameter of the universe and the wave number and mass of the field.

Equation of motion

We consider the 2-dimensional metric

$$ds^2 = a^2(\eta)\left[-d\eta^2 + dx^2\right] \tag{1}$$

with scalar factor $a(\eta) = -(H\eta)^{-1}$ and η cosmological time. Through separation of temporal and spatial dependencies, the Klein-Gordon equation in curved spacetime gives the dynamical equation

$$\phi_k(\eta)'' + \left(k^2 + m^2a^2(\eta) - \frac{a''(\eta)}{a(\eta)}\right)\phi_k(\eta) = 0 \tag{2}$$

for the modes of the (scaled) massive scalar field. We can use the Hamiltonian formalism writing the field mode and conjugated momenta in terms of the creation and annihilation operators,

$$\phi_k = \frac{1}{\sqrt{2\omega_k}}(a_k + a^\dagger_{-k}) \tag{3}$$

$$\pi_k = i\sqrt{\frac{\omega_k}{2}}(a^\dagger_k - a_{-k}) \tag{4}$$

where ω_k comes from the dispersion condition containing the dependency from the expansion

$$\omega_k(\eta)^2 = k^2 + m^2_{eff.} = k^2 + m^2a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \tag{5}$$

The time-dependent creation-annihilation operators in the asymptotic future are related to the ones operating on the initial state in η_0 through the Bogolubov relations

$$\begin{pmatrix} \hat{a}_k(\eta) \\ \hat{a}^\dagger_{-k}(\eta) \end{pmatrix} = \begin{pmatrix} u_k(\eta) & v_k(\eta) \\ v^*_k(\eta) & u^*_k(\eta) \end{pmatrix} \begin{pmatrix} \hat{a}_k(\eta_0) \\ \hat{a}^\dagger_{-k}(\eta_0) \end{pmatrix} \tag{6}$$

where η_0 is the specific time chosen to set the vacuum state defined by

$$a_k(\eta_0)|0,\eta\rangle = 0 \tag{7}$$

We can obtain the expression for $u_k(\eta)$ and $v_k(\eta)$ by imposing the condition that the Heisemberg equation of motion for the operators must be satisfied

$$\hat{a}'_k(\eta) = i\left[\hat{a}_k, \hat{H}\right] \tag{8}$$

from which follows

$$u' = -i\omega_k u + \frac{a'}{a}v^* \tag{9}$$

$$v' = -i\omega_k v + \frac{a'}{a}u^* \tag{10}$$

The evolution of the vacuum state can be expressed in the asymptotic future basis of the Fock space as

$$|0,\eta\rangle = \frac{1}{|u_k|}\sum_{n=0}^{\infty}\left(\frac{v_k}{u^*_k}\right)^n |n_k, n_{-k}\rangle \tag{11}$$

Tracing out -k modes, this leads to the reduced density matrix

$$\rho(\eta) = \sum_{n=0}^{\infty}\lambda_n|n_k\rangle\langle n_k| \ , \ \lambda_n = \frac{1}{|u_k|^2}\left|\frac{v_k}{u_k}\right|^{2n} \equiv \frac{1}{|u_k|^2}\gamma^n \tag{12}$$

The Bogolubov coefficient can be expressed using to the auxiliary functions

$$f = \frac{1}{\sqrt{2\omega_k}}(u + v^*) \quad g = \sqrt{\frac{\omega_k}{2}}(u - v^*) \tag{13}$$

which satisfy

$$f(\eta) = \sqrt{\eta} H_{\nu}^{(2)}(\eta) \tag{14}$$

where $H_{\nu}^{(2)}$ is the Hankel function of the second kind with order $\nu = \frac{1}{2}(9 - 4m^2/H^2)^{1/2}$, while

$$g(\eta) = i\left(f'(\eta) - \frac{a'}{a} f(\eta)\right) \tag{15}$$

Entanglement Entropy and Quantum Fisher Information

The entanglement entropy for the asymptotic state is

$$S = -\sum_{n=0}^{\infty} \lambda_n ln(\lambda_n) = 2\frac{ln(|u|)}{|u|^2}\frac{1}{1-\gamma} - \frac{ln(\gamma)}{|u|^2}\frac{\gamma}{(1-\gamma)^2} \tag{16}$$

while the quantum Fisher Information associated to the generic variable θ as

$$F_Q(\theta) = \sum_{n=0}^{\infty} \lambda_n (\partial_\theta ln \lambda_n)^2 \tag{17}$$

These quantities can be used to describe the amount and distribution of information that can be extracted from the system about its dynamics, in terms of fundamental variables involved. They could allow, for example, to determine whether a particular range in the value of the wave number or mass of the scalar field exists that optimises the precision of the universe expansion measurement. Also, the dependency of the evolution of the entanglement from such variables may lead to interesting observations in the study of transitions from the quantum to the classical domain.

Reference

A. Duncan, Phys. Rev. D 17, 964 (1978)

J.Wang Z.Tian J. Jing, H. Fan, Parameter estimation for an expanding universe, Nucl.Phys. B 892 (2015) 390-399

I.Fuentes, R.B.Mann, E.Matrin-Matrinez, S.Moradi, Entanglement of Dirac fields in an expanding spacetime, arXiv:1007.1569v2 [quant-ph] 2010