CMBの温度･偏光揺らぎにおける弱い重力レンズ効果:再構築法の開発

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1. Introduction

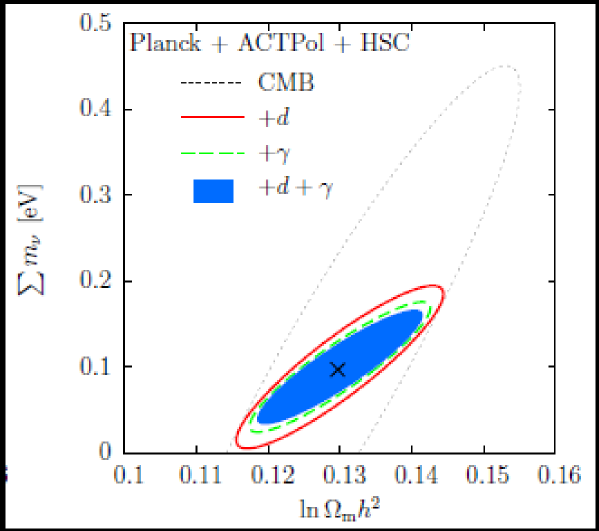
1.1 What can we probe with CMB lensing

1.2 Current measurement of CMB-lensing power spectrum

1.3 Purpose of our work

What can we probe with CMB lensing ?

- ➢ **Weak lensing as a probe of dark energy, massive neutrinos, …**
  - ✓ **Sensitive to high-z structure**
  - ✓ **Properties of source (CMB) are well known**
  - ✓ **Complementary to other probes**



***TN*, Saito, Taruya '10**

- ➢ **Primordial gravitational wave**
  - ✓ **On small scales, B-mode is dominated by lensing.**
  - ✓ **Constraints on r would be improved by extracting lensing B-mode.**

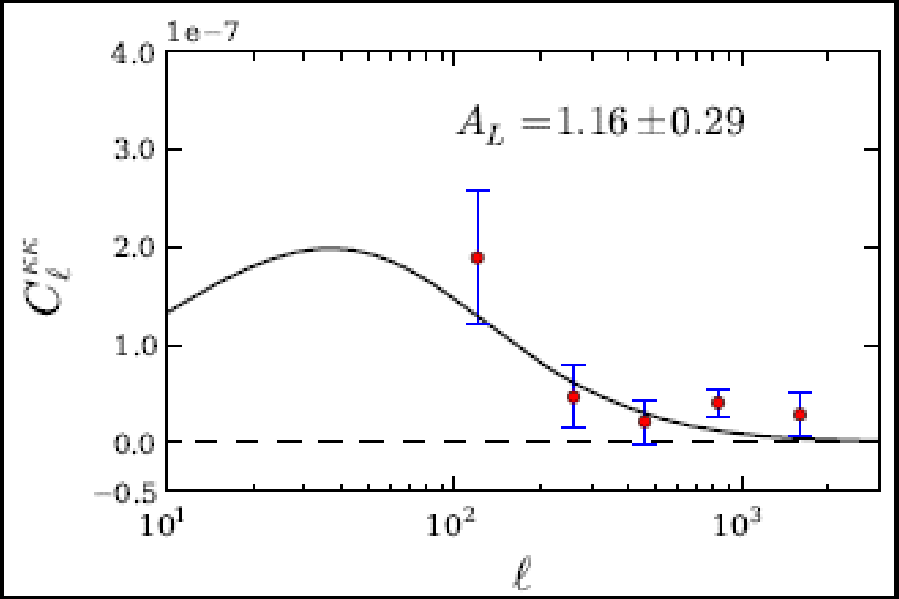
- ➢ **Some sources which induce Curl-type deflection angle**
  - ✓ **Cosmic string, gravitational wave, …**

Current detection of CMB lensing PS

➢ **detection**

$C_\ell^{g\kappa}$
- Smith+ ’07 (3.4σ)
- Hirata+ ’08 (2.5σ)

$C_\ell^{\kappa\kappa}$
- Smidt+ ’10 (~2σ)
- Das+ ’11 (~4σ)



Das+ ’11

➢ **Upcoming, future experiments**

✓ **Ground**
- PolarBear (2011-)
- ACTPol (2012-)

✓ **Space**
- Planck (2010-)
- CMBPol (?)

With these upcoming experiments, CMB lensing would be detected with high precisions enough to give us several cosmological implications

Motivation of our work

Deflection angle

- ✓ If we consider the lensing effect arising from the linear matter density fluctuations, the deflection angle is related to the lensing potential as

$$d_i(\vec{n}) = \partial_i \phi(\vec{n})$$

- ✓ This relation is assumed in several reconstruction methods

(e.g., Hu & Okamoto '02)

Curl-type deflection angle

- ✓ In general, deflection angle has two components

2D Levi-Chivita tensor

$$d_i(\vec{n}) = \underbrace{\partial_i \phi(\vec{n})}_{\text{Gradient part}} + \underbrace{\epsilon_{ij}\partial_j \omega(\vec{n})}_{\text{Curl part}}$$

Gradient part — Curl part

- ✓ Curl-mode is non-zero if the lensing effect is induced by vector/tensor metric perturbations (e.g., cosmic string, primordial gravitational wave)

To probe physics generating the curl mode, we need a method for reconstructing curl mode from observational data

Purpose 1

Find an algorithm for reconstructing deflection angle including both gradient and curl mode

- **Previous works which consider curl-type deflection angle**
  - Hirata & Seljak '03 ▪Based on the likelihood estimator
  - Cooray+ '05 ▪Based on the optimal quadratic estimator proposed by Hu & Okamoto '02
- **Our work**
  - ✓ Our estimator is based on Okamoto & Hu '03 (OH03), but including curl-type deflection angle (extension of Cooray+'05 in full sky)
  - ✓ Then, we show that the gradient- and curl-type deflection angle can be reconstructed with unbiased condition

Purpose 2

- **Sources of curl-type deflection angle**

An example: cosmic string

- ✓ **Cosmic string can be produced by the phase transition in the early universe**

- ✓ **The primordial CMB temperature anisotropies produced by cosmic strings are less than ~10% (corresponds to a constraint on dimensionless string tension: $G\mu < O(10^{-7})$)**

(e.g., Wyman+'05, Seljak+'06, Bevis+ '07)

- ✓ **Cosmic string induces vector/tensor perturbations and would produce curl-type deflection angle : cosmic string would be constrained from curl mode**

Estimate expected detectability of cosmic string by reconstructing curl-type deflection angle

2. Brief review of Okamoto & Hu 2003

Brief Review of OH'03

➢ Definition of estimator

$$\hat{\phi}_{\ell m}^{(XY)} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} f_{\ell L_1 L_2}^{XY} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

where $\tilde{X}_{\ell m}$ **and** $\tilde{Y}_{\ell m}$ **is** $\tilde{\Theta}_{\ell m}$, $\tilde{E}_{\ell m}$, **or** $\tilde{B}_{\ell m}$

To determine the functional form of *f* theoretically, the following conditions are imposed :

1. Unbiased

Ensemble average over the estimator $\hat{\phi}_{\ell m}^{XY}$ ***with fixing the lensing potential*** **should be equals to the lensing potential**

$$\langle \hat{\phi}_{\ell m}^{XY} \rangle_{CMB} = \phi_{\ell m}$$

2. Optimal

Choosing *f* so that N_ℓ **is minimized**

$$\left\langle \hat{\phi}_{\ell m}^{(XY)} \left(\hat{\phi}_{\ell m}^{(XY)} \right)^* \right\rangle = \underline{N_\ell^{\phi,(XY)}} + C_\ell^{\phi\phi}$$

Brief Review of OH'03

➢ Functional form of *f*

- ✓ described by the observed (lensed) power spectra, $\hat{C}_\ell^{XY}$, and unlensed Cl's

$$f^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{F^{XY}_{\ell L_1 L_2}}{[\Phi F]^{XY}_\ell}$$

Summation : $\sum_{L_1}\sum_{L_2}\Phi^{XY}_{\ell L_1 L_2}F^{XY}_{\ell L_1 L_2}$

$$F^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Phi^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Phi^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

* The quantity Φ depends on unlensed Cl's

➢ Reconstruction

Observed anisotropies $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}, \tilde{B}_{\ell m}$

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1}\sum_{L_2 M_2} f^{XY}_{\ell L_1 L_2}\begin{pmatrix}\ell & L_1 & L_2\\ -m & M_1 & M_2\end{pmatrix}\tilde{X}_{L_1 M_1}\tilde{Y}_{L_2 M_2}$$

- ✓ In principle, we can reconstruct the lensing potential from observed CMB maps.

3. Estimator

3.1 Lensing fields as a quadratic statistics

3.2 Definition of our estimator

3.3 Weight function

Lensing field as a quadratic statistics

- **Average with fixed lensing fields**

Similar analogy to OH'03

$$\langle \tilde{\Theta}_{L_1M_1}\tilde{\Theta}_{L_2M_2}\rangle_{CMB} = C_{L_1}^{\Theta\Theta}\delta_{L_1L_2}\delta_{M_1M_2}(-1)^{M_1} + \sum_{\ell m}(-1)^m[\Phi^{\Theta\Theta}_{\ell L_1 L_2}\phi_{\ell m} + \Omega^{\Theta\Theta}_{\ell L_1 L_2}\omega_{\ell m}]\begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix}$$

[Key property]

$$\Phi_{\ell L_1 L_2} = 0\ ,\ \textbf{for } \ell + L_1 + L_2 = \textbf{odd}$$

$$\Omega_{\ell L_1 L_2} = 0\ ,\ \textbf{for } \ell + L_1 + L_2 = \textbf{even}$$

➡ **$\phi_{\ell m}$, $\omega_{\ell m}$ are expressed independently**

For ω,

$$\omega_{\ell m} = (2\ell+1)(-1)^m \times \sum_{L_1M_1}\sum_{L_2M_2}\frac{G^{\Theta\Theta}_{\ell L_1 L_2}}{[\Omega G]^{\Theta\Theta}_{\ell}}\begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix}\langle \tilde{\Theta}_{L_1M_1}\tilde{\Theta}_{L_2M_2}\rangle_{CMB}$$

Arbitrary function (→ $G^{\Theta\Theta}_{\ell L_1 L_2}$)

Estimator including curl mode

➢ Definition of estimators

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} f^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

$$\hat{\omega}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} g^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

where $\tilde{X}_{\ell m}$ and $\tilde{Y}_{\ell m}$ is $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}$, or $\tilde{B}_{\ell m}$

To determine the functional form of f and g theoretically, the following conditions are imposed :

1. Unbiased

Note: In Cooray +'05, they claim their estimator is not satisfied this condition, but I checked their flat sky estimator also satisfies the condition.

Ensemble average over the estimators $\hat{\phi}^{XY}_{\ell m}$ and $\hat{\omega}^{XY}_{\ell m}$ *with fixing the lensing fields* should be equals to the lensing fields, respectively

$$\langle \hat{\phi}^{XY}_{\ell m} \rangle_{CMB} = \phi_{\ell m} \qquad \langle \hat{\omega}^{XY}_{\ell m} \rangle_{CMB} = \omega_{\ell m}$$

2. Optimal

Choosing f and g so that N_ℓ is minimized

$$\left\langle \hat{\phi}^{(XY)}_{\ell m} \left(\hat{\phi}^{(XY)}_{\ell m} \right)^* \right\rangle = \underline{N^{\phi,(XY)}_\ell} + C^{\phi\phi}_\ell \qquad \left\langle \hat{\omega}^{(XY)}_{\ell m} \left(\hat{\omega}^{(XY)}_{\ell m} \right)^* \right\rangle = \underline{N^{\omega,(XY)}_\ell} + C^{\omega\omega}_\ell$$

Estimator including curl mode

➢ Functional form of f and g

- ✓ Both f and g are described by the observed (lensed) and unlensed Cl's

- ✓ Thanks to the property of parity, the estimators, $\hat{\phi}^{XY}_{\ell m}$ and $\hat{\omega}^{XY}_{\ell m}$ are reconstructed in a similar way that of OH'03, and f is the same as that of OH'03

- ✓ The functional form of g is similar to that of f

$$f^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{F^{XY}_{\ell L_1 L_2}}{[\Phi F]^{XY}_{\ell}} \qquad F^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Phi^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Phi^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

$$g^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{G^{XY}_{\ell L_1 L_2}}{[\Omega G]^{XY}_{\ell}} \qquad G^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Omega^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Omega^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

*** The quantity Ω depends on unlensed Cl's but the dependence is different from Φ**

Summary of Our Estimator

Observed anisotropies $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}, \tilde{B}_{\ell m}$

$$G^{XY}_{\ell L_1 L_2} = \frac{\hat{\mathcal{C}}^{XX}_{L_2}\hat{\mathcal{C}}^{YY}_{L_1}\Omega^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{\mathcal{C}}^{XY}_{L_1}\hat{\mathcal{C}}^{XY}_{L_2}\Omega^{XY}_{\ell L_2 L_1}}{\hat{\mathcal{C}}^{XX}_{L_1}\hat{\mathcal{C}}^{YY}_{L_2}\hat{\mathcal{C}}^{XX}_{L_2}\hat{\mathcal{C}}^{YY}_{L_1} - \left(\hat{\mathcal{C}}^{XY}_{L_1}\hat{\mathcal{C}}^{XY}_{L_2}\right)^2}$$

$$g^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{G^{XY}_{\ell L_1 L_2}}{[\Omega G]^{XY}_{\ell}}$$

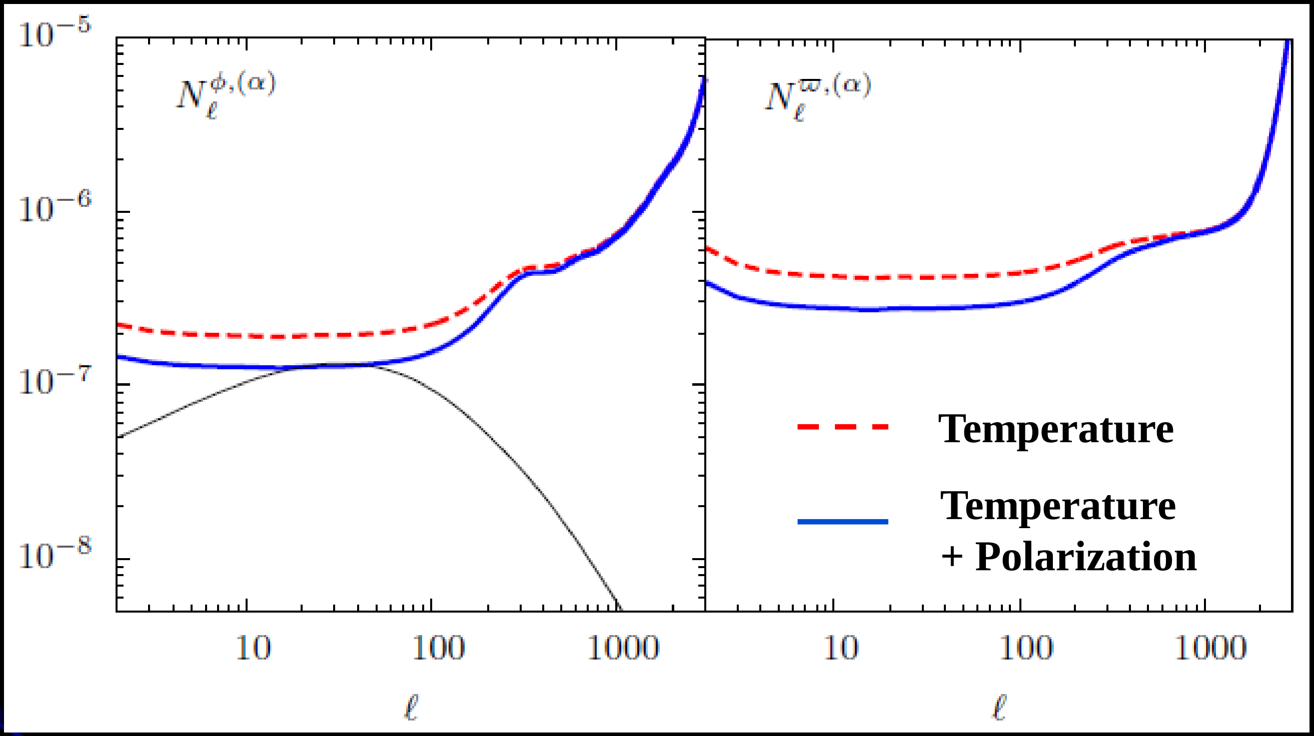
$$\omega^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1}\sum_{L_2 M_2} g^{XY}_{\ell L_1 L_2}\begin{pmatrix}\ell & L_1 & L_2\\ -m & M_1 & M_2\end{pmatrix}\tilde{X}_{L_1 M_1}\tilde{Y}_{L_2 M_2}$$

4. Numerical Calculation

4.1 Noise Spectra

4.2 Implications

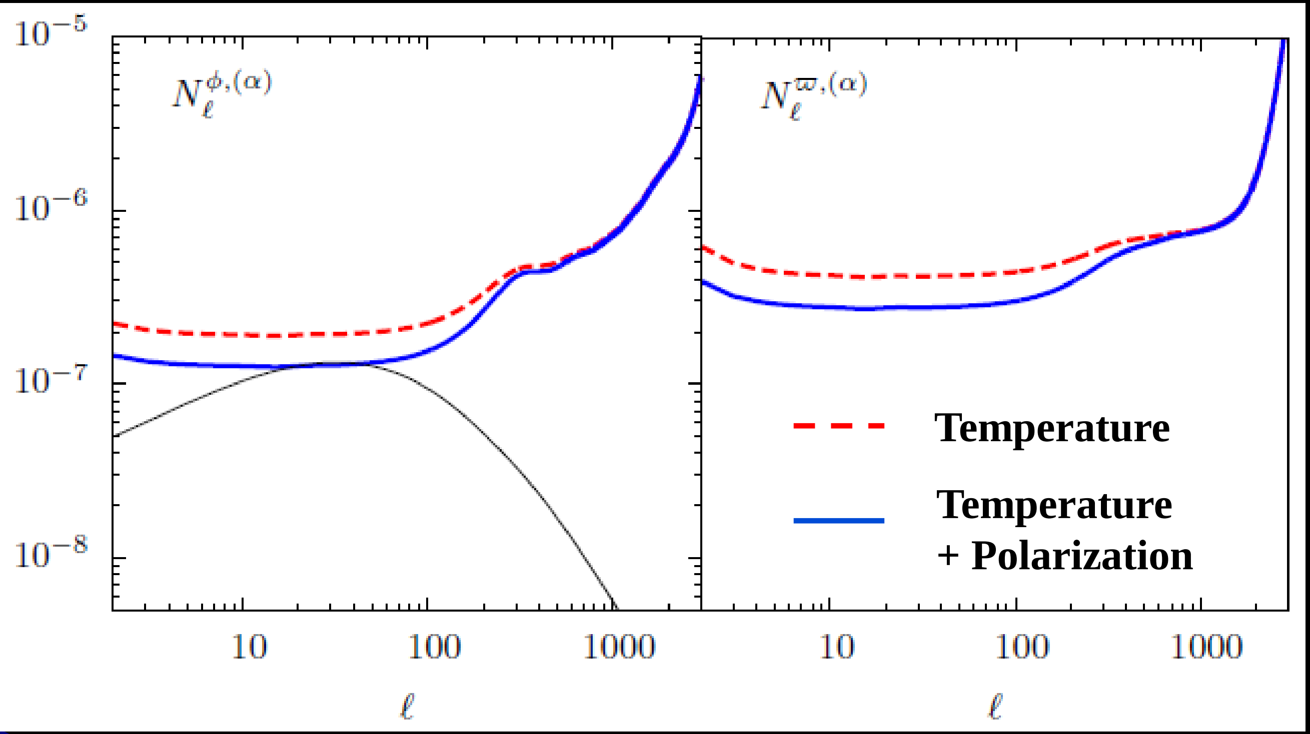
Noise Spectra : Planck



Note: For ACTPol, the noise improvement by including polarization is significant compared to that of Planck

The noise of curl mode is comparable to that of gradient mode

Noise Spectra : ACTPol



Note 1: For ACTPol, the noise improvement by including polarization is significant compared to that of Planck

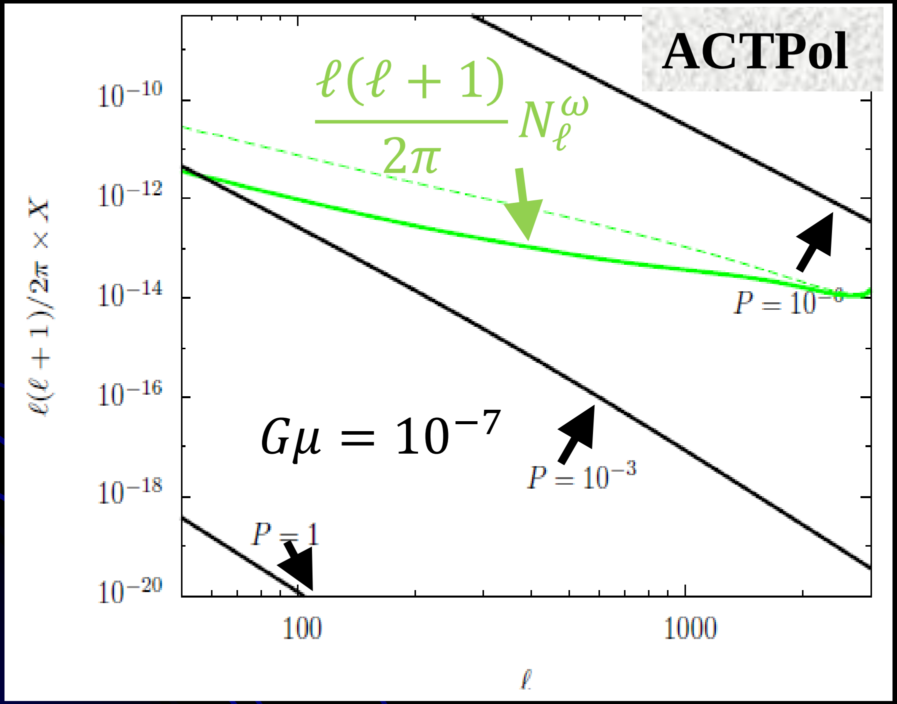
Note 2: The noise of curl mode is comparable to that of gradient mode

Implications for cosmic strings

➢ Assumptions

- ✓ Nambu-string
- ✓ VOS model (Martins+'02)
- ✓ Energy loss rate (Martins+'02,'04) $\sim 0.23 P v_{rms} \rho_{str}/\xi$
- ✓ Number of string in the region $[z, z+\delta z]$ is $\delta z(\frac{dV}{dz})/\xi^3$
- ✓ Straight string

➢ Results



$$\ell^2 C_\ell^{\omega\omega} \propto (G\mu)^2 P^{-\frac{5}{2}} \ell^{-5}$$

If $P < 10^{-3}$ and $G\mu > 10^{-7}$, the curl-type deflection angle induced by cosmic string would be detected

Summary

- ✓ **We show an algorithm for reconstructing deflection angle including both gradient and curl mode**

  Then, thanks to property of parity, the gradient and curl mode can be reconstructed similar to that of OH'03.

- ✓ **Assuming ACTPol, we roughly estimate the expected constraint on cosmic string using the curl mode.**

  Using ACTPol data, if $G\mu > O(10^{-7})$ and $P < O(10^{-3})$, the curl-type deflection angle from cosmic string would be detected

Curl mode has no contribution from liner-matter density fluctuations, so in this respect, considered as pure signal of string, which is an advantage of this method compared to other probes of string