Radiation spectra from relativistic electrons moving in turbulent magnetic fields

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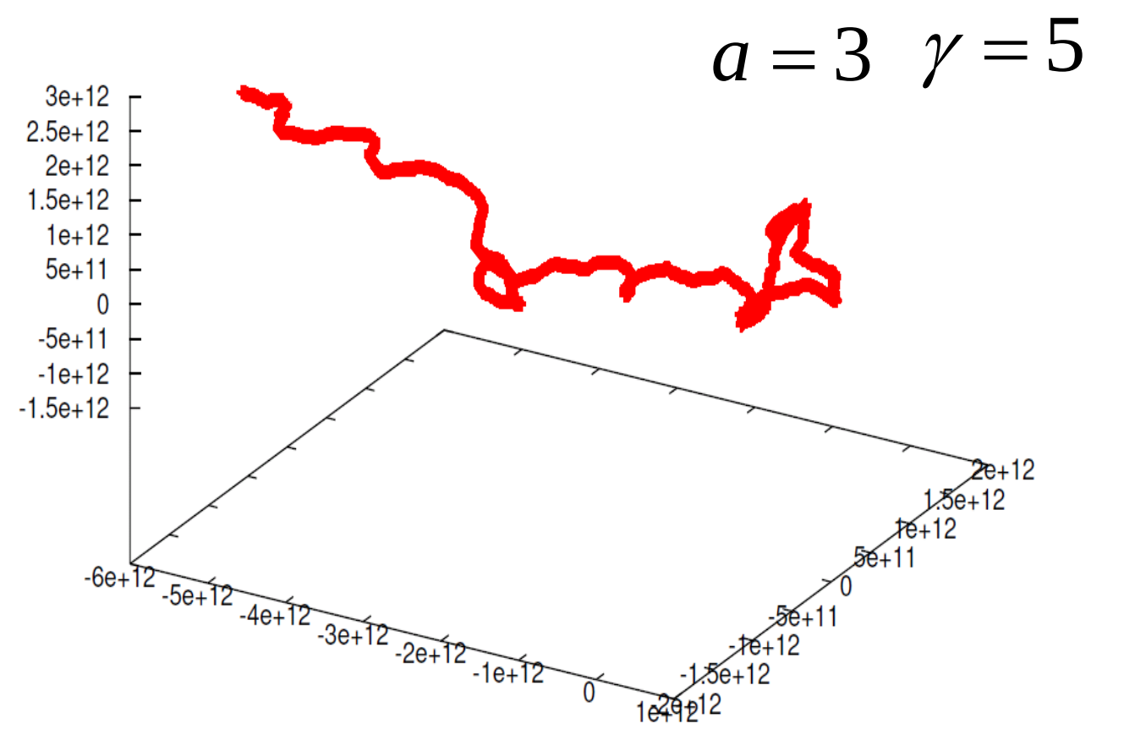
Example of trajectory

$a=3 \quad \gamma=5$

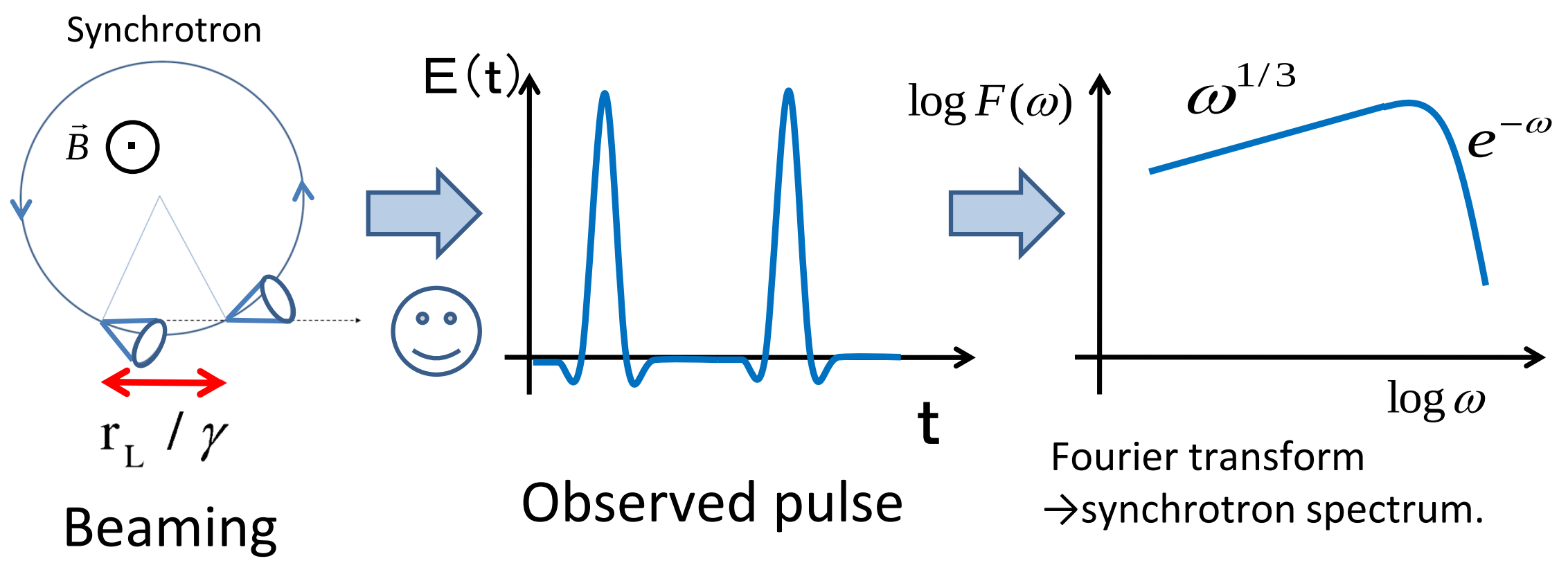


Abstract

We numerically calculate the radiation spectra from relativistic electrons moving in small scale turbulent fields expected in high energy astrophysical sources. They are characterized by the strength parameter $a=\lambda_B e|B|/mc^2$, where λ_B is the length scale of the turbulent field. We perform numerical calculations for several values of a with $\gamma=10$. We obtain various types of spectra ranging between jitter radiation and synchrotron radiation. For $a\approx 7$, the spectrum turns out to take a novel shape which has not been noticed before. It is like a synchrotron spectrum in the middle frequency region, but in the low frequency region it is a broken power law and in the high frequency region an extra power law component appears beyond the synchrotron cutoff. We give a physical explanation of these features.

Introduction

The synchrotron theory is based on the assumption that the scale of magnetic field is much larger than the Larmor radius of radiating electrons.



When the magnetic field is highly turbulent on small scales, electrons suffer from random accelerations and do not trace a helical trajectory. The radiation spectrum is characterized by the strength parameter

$$a=\lambda_B e|B|/mc^2$$

The characteristic scale of magnetic fields generated near the shock front is order of skin depth as predicted by the analysis of Weibel instability. Then, the scalelength of turbulent magnetic field is described by using a coefficient as κ

$$\lambda_B=\kappa\frac{c}{\omega_{pe}\Gamma_{bulk}}$$

Where ω_{pe} is the plasma frequency, Γ_{bulk} is the Lorenz factor of colliding shells.

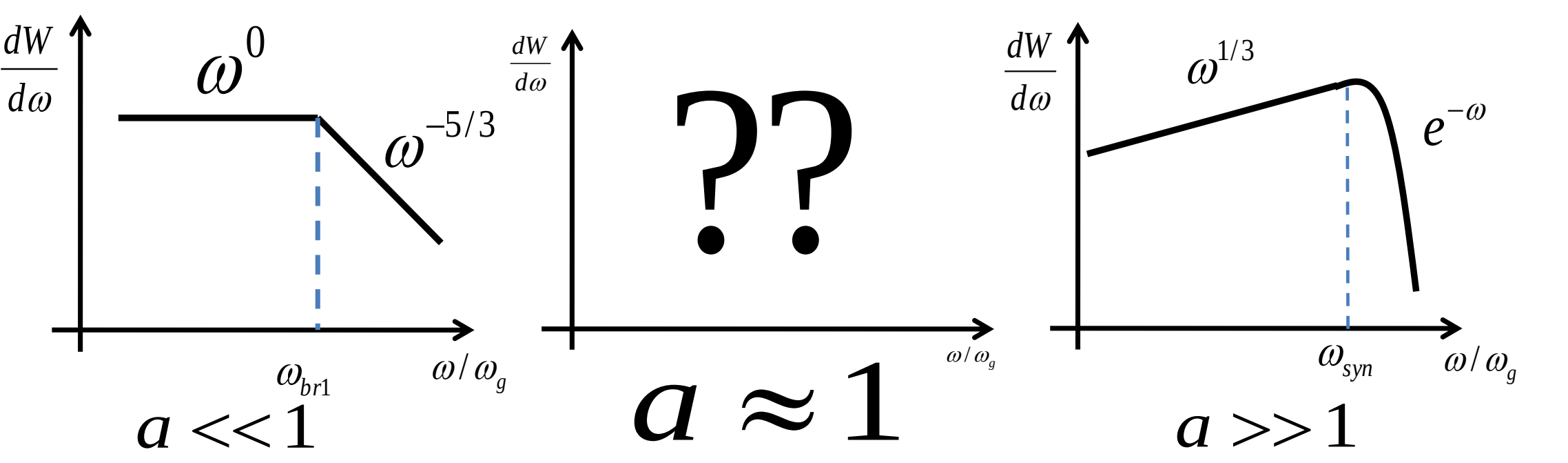
The energy conversion rate into the magnetic fields is

$$\varepsilon_B=\frac{B^2/8\pi}{\Gamma_{\rm int} n m_e c^2}$$

$\boxed{\kappa\approx 10 \quad \varepsilon_B\approx 0.1}$: typical value from PIC simulations.

$$\downarrow$$

$$\boxed{a\equiv\frac{\lambda_B}{r_L/\gamma}=\kappa\sqrt{\frac{2\gamma_{cold}\varepsilon_B}{\Gamma_{\rm int}}}\approx \underline{O(1)}}$$



Jitter radiation

synchrotron radiation

In this work, we study the intermediate regime.

Method

3D turbulent magnetic field

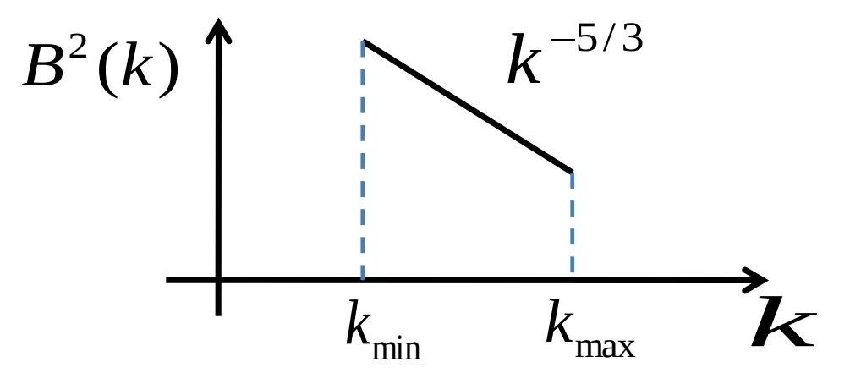
$$\vec{B}(\vec{x})=\sum_{n=1}^{N}A_n\exp\{i(\vec{k}\cdot\vec{x}+\beta_n)\}\hat{\xi_n}$$

$$\hat{\xi_n}=\cos\psi_n\vec{e_x'}+i\sin\psi_n\vec{e_y'}$$

$$\vec{e_z'}=\frac{\vec{k_n}}{k_n}$$

$$A_n^2=\sigma^2 G_n\left[\sum_{n=1}^{N}G_n\right]^{-1}$$

$$G_n=\frac{4\pi k_n^2\Delta k_n}{1+(k_nL_c)^{\frac{11}{3}}}$$



σ mean value of B
$\sigma^2=<B^2>$

$k_{\rm max}=k_{\rm min}\times 100$
$\lambda_{\rm max}=2\pi/k_{\rm min}$

Define a by $\lambda_{\rm max}$

$$\boxed{a\equiv\frac{\lambda_{\rm max}}{r_L/\gamma}=\frac{2\pi}{k_{\rm min}}\frac{e\sigma}{m_ec^2}}$$

Solve the equation of motion

$$\gamma m_e\frac{d\vec{v}}{dt}=-e\vec{\beta}\times\vec{B}$$

Radiation spectrum is calculated using Lienard-Wiechert potential.

$$\frac{dW}{d\omega d\Omega}=\frac{e^2}{4\pi c^2}\left|\int_{-\infty}^{\infty}dt'\frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\vec{n})^2}\exp\{i\omega(t'-\frac{\vec{n}\cdot\vec{r}(t')}{c})\}\right|^2$$

$\vec{n}$ Unit vector points observer

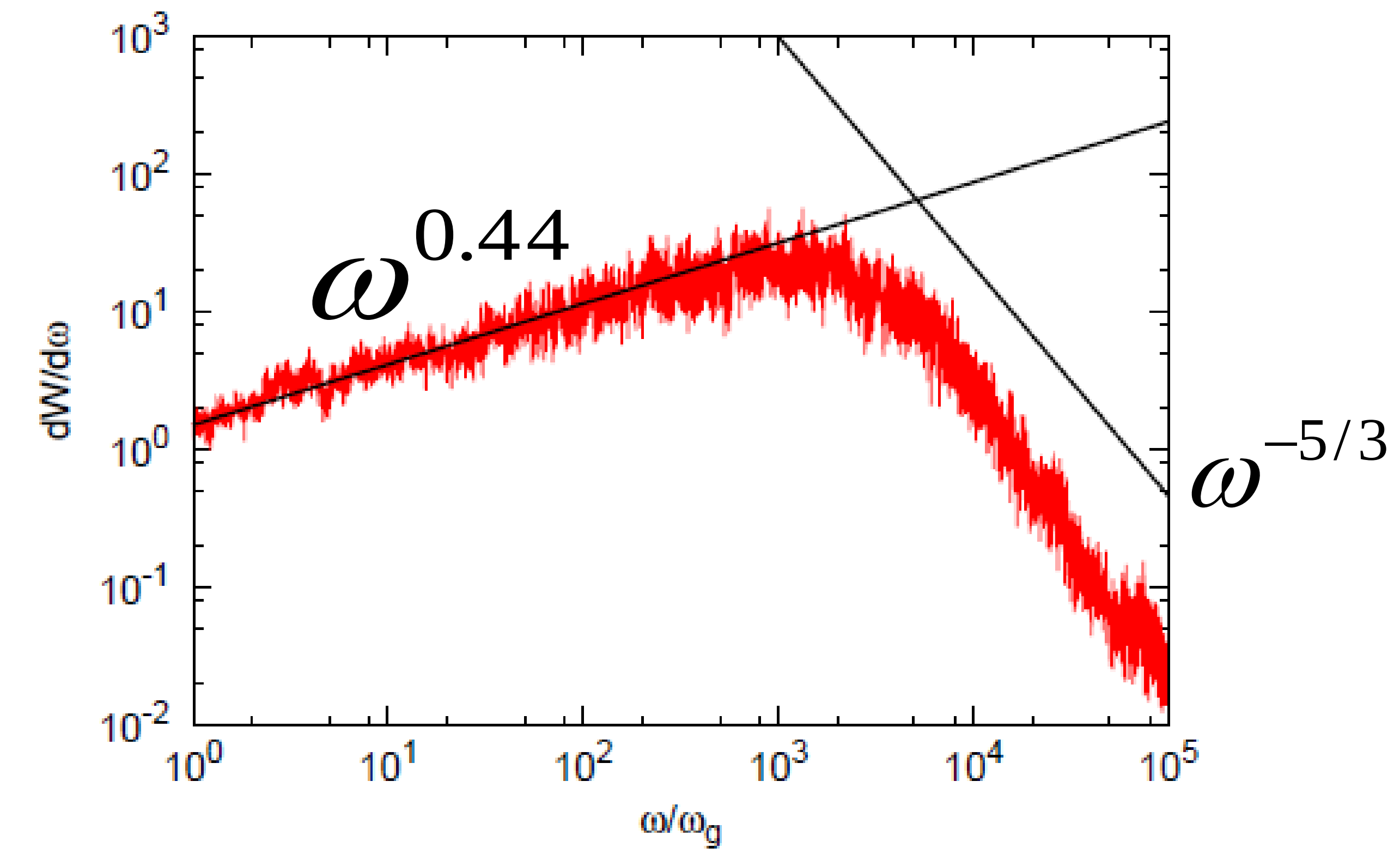
t' Retarded time

Results $[\gamma=10]$

The frequency is normalized by the fundamental frequency $\omega_g=e\sigma/\gamma mc$ and the magnitude is arbitrarily scaled.

The color lines are calculated spectrum.

$a=3$

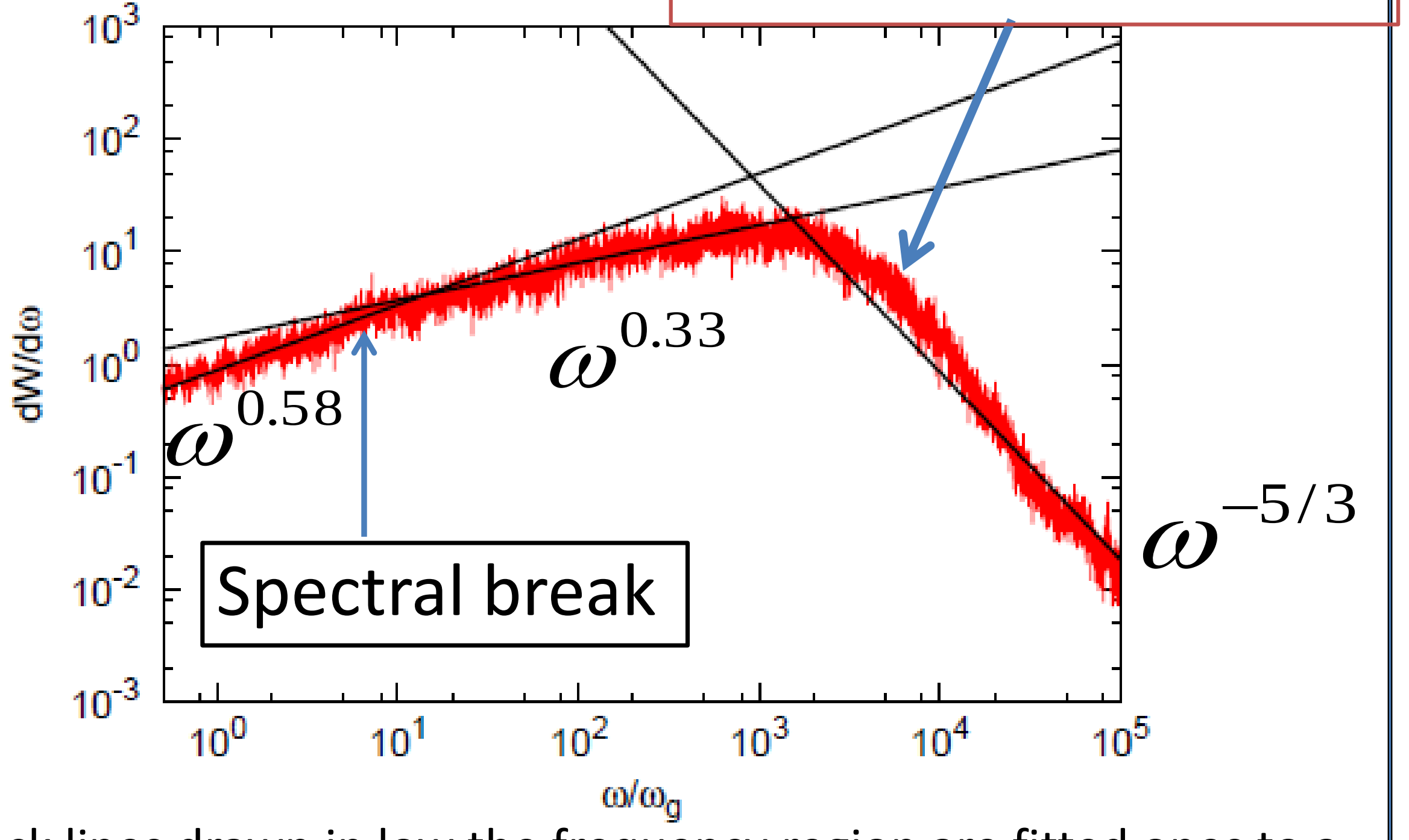


Radiation spectrum for $a=3$ and $\gamma=10$

The black line in the low frequency region shows a power law spectrum with an index 0.44. The black line in the high frequency region is $dW/d\omega\propto\omega^{-5/3}$ for reference. Power law index of low frequency spectrum is harder than the synchrotron theory predicts. This spectral feature can be explained by an extrapolation of DSR (jitter) for $a<1$.

$a=5$

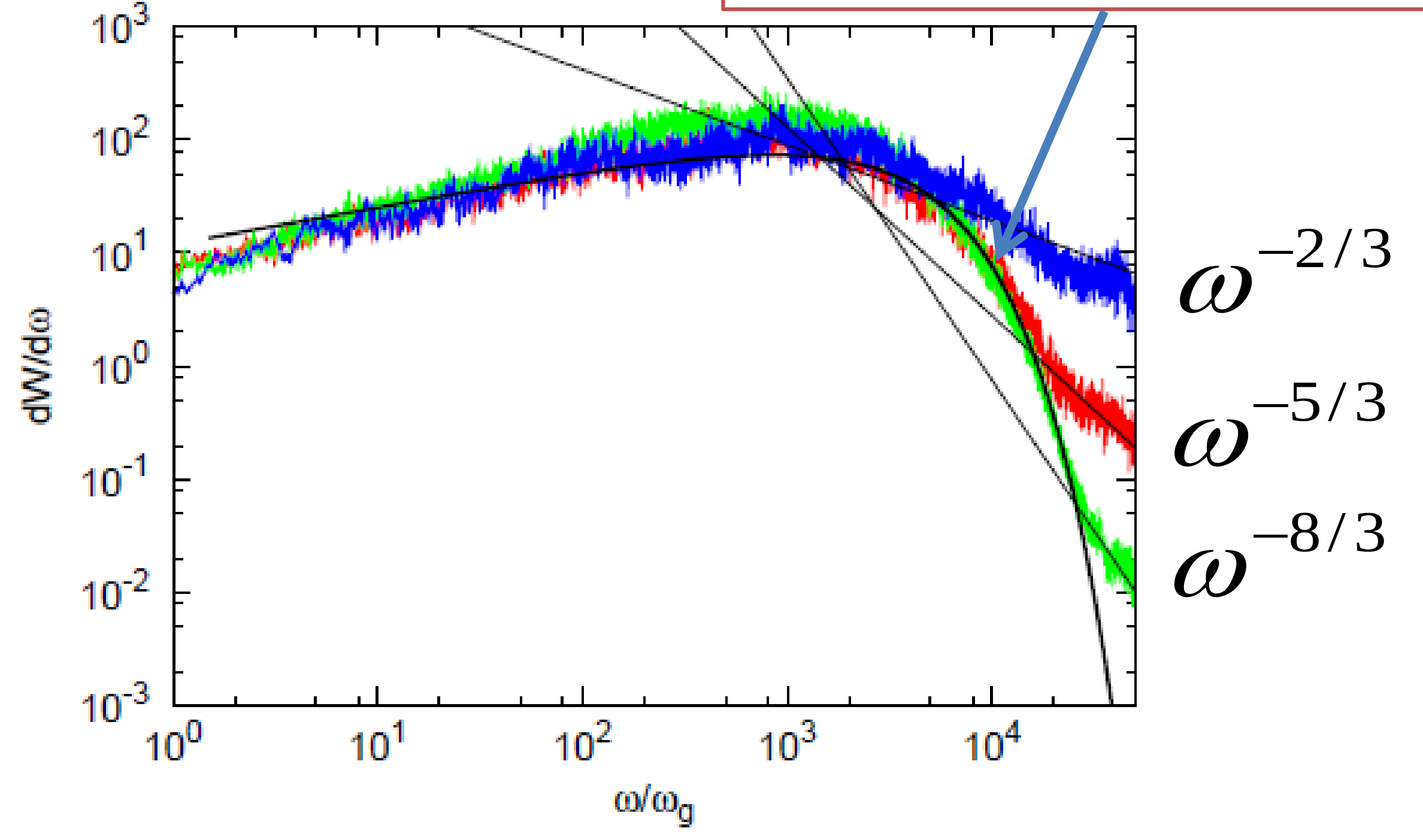
Exponential cutoff ?



Two black lines drawn in low the frequency region are fitted ones to a power law spectrum in the range of 0.5-10, and 10-1000, respectively. We see a broad hump in the peak region, which is identified with the synchrotron spectrum with an exponential cutoff.

$a=7$

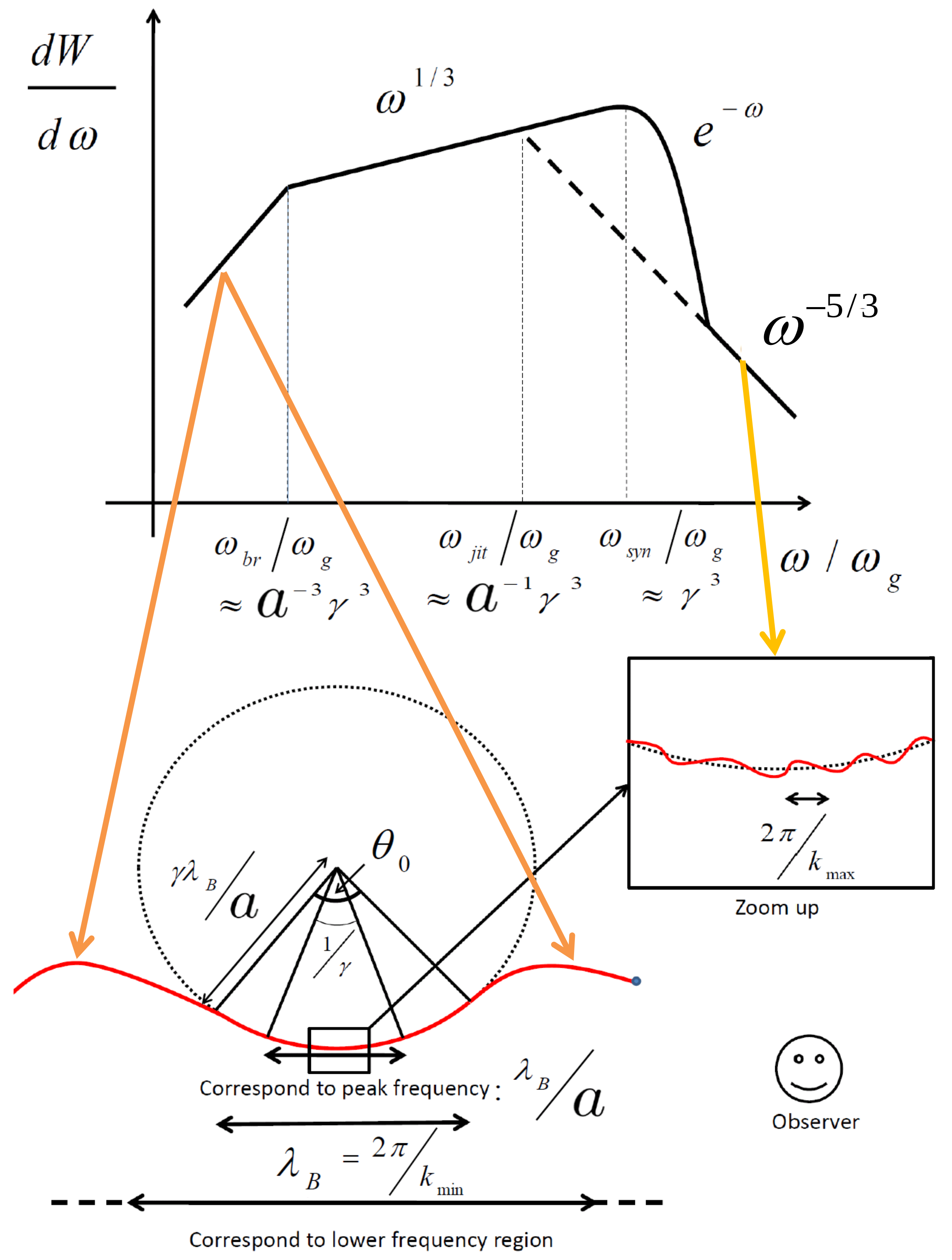
Exponential cutoff !!



The spectra for $a=7$, for three different values of the power law index of turbulent magnetic fields. The curved black line is a theoretical curve of synchrotron radiation. We see effects of difference of the turbulent magnetic fields in the high energy region.

Interpretation

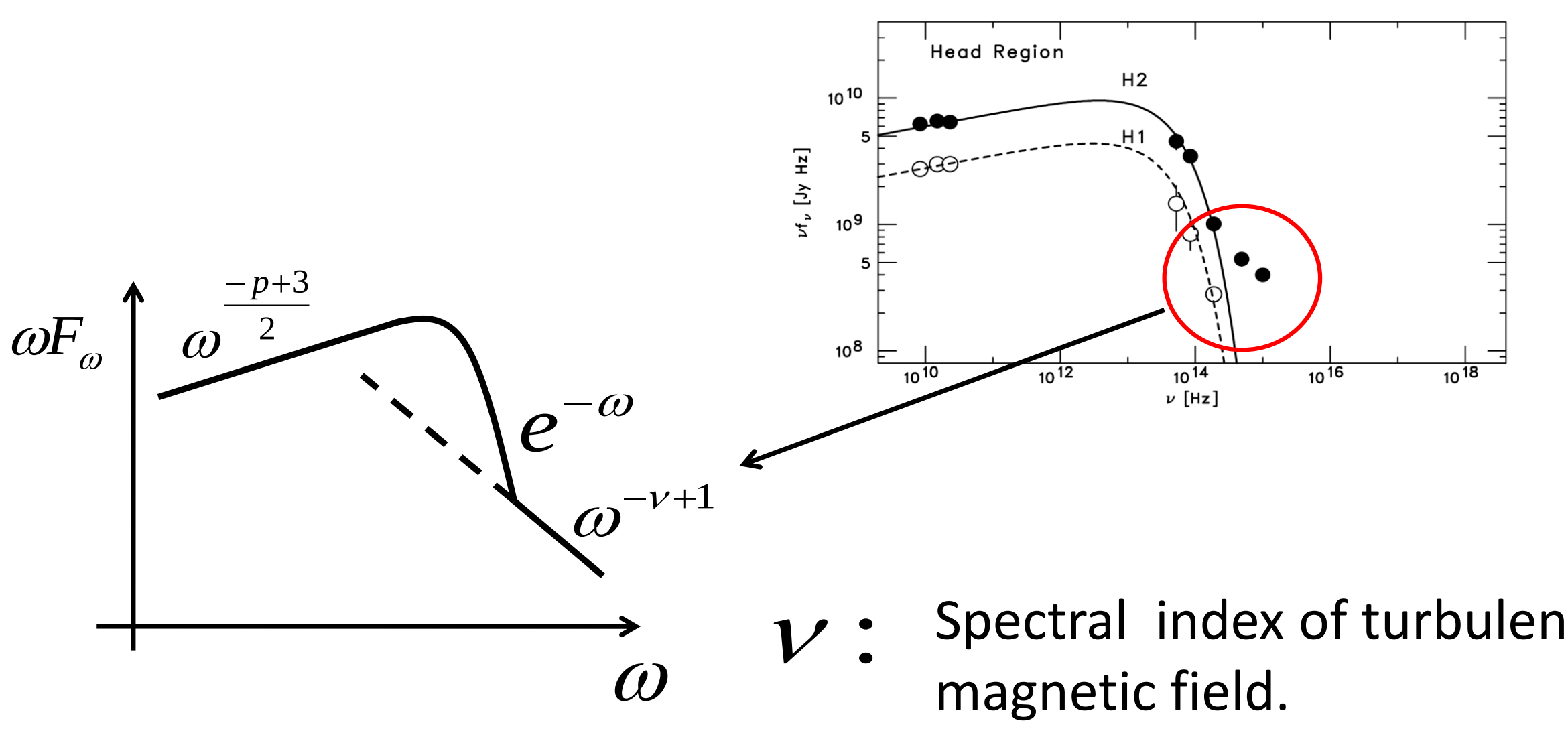
We interpret the spectral feature for $a=5$ and $a=7$. The conceptual diagram of these spectra for $5<a<\gamma$ and a schematic picture of an electron trajectory is depicted below.



The radius of the guiding circle is the Larmor radius $r_L=\gamma\lambda_B/a$. Low frequency photons are emited from the motion on scales larger than λ_B The spectral break at $a^{-3}\gamma^3$ corresponds to the break of synchrotron approximation at the scale of λ_B.
The scale $\lambda_B/a=r_L/\gamma$ corresponds to the peak frequency.
On the smallest scale down to $2\pi/k_{\rm max}$, the trajectory is approximately straight, and jittering is responsible for the power law component in the highest frequency region.

Conclusions

We confirm that the spectrum for $a\approx 3$ is a broken power law with the low energy spectral index harder than the synchrotron theory predicts. Furthermore, we find that the spectrum for $a\approx 7$ takes a novel shape described by a superposition of a broken power law spectrum and a synchrotron one. Especially, an extra power law component appears beyond the synchrotron cutoff in the high frequency region. This novel spectral shape may be seen in various scenes. For example, the spectrum of 3C273 jet at the knot region may be due to this feature.



ν : Spectral index of turbulent magnetic field.