高次元動的時空における漸近構造と角運動量

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1. Introduction

漸近的に平坦な時空の無限遠

- spatial infinity　　$r = \infty$

  時空はほぼ定常的。 ADM質量、ADM角運動量等。

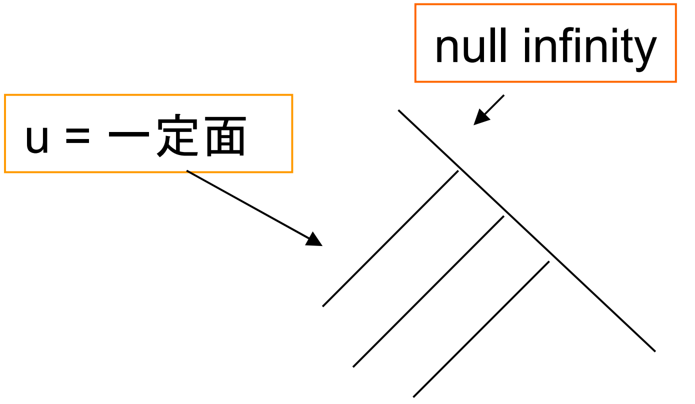
  → BHの唯一性定理

- null infinity　　$t = \infty,\ r = \infty,\ t - r = \text{finite}$

  時空は動的。 Bondi質量等。

ここでは 5 次元時空におけるnull infinity を考える。

2. Bondi coordinate



Bondi 座標

$$x^a = (u, r, \theta, \phi, \psi)$$

計量

$$ds^2 = -\frac{Ve^B}{r^2}du^2 - 2e^B du dr + r^2 h_{AB}(dx^A + U^A du)(dx^B + U^B du)$$

$$h_{AB} = \begin{pmatrix} e^{C_1} & \sin\theta\sinh D_1 & \cos\theta\sinh D_2 \\ \sin\theta\sinh D_1 & e^{C_2}\sin^2\theta & \sin\theta\cos\theta\sinh D_3 \\ \cos\theta\sinh D_2 & \sin\theta\cos\theta\sinh D_3 & e^{C_3}\cos^2\theta \end{pmatrix}$$

ゲージ条件

$$\det h_{AB} = \sin^2\theta\cos^2\theta$$

Gravitational fields near null infinity

$$h_{AB} = \begin{pmatrix} e^{C_1} & \sin\theta\sinh D_1 & \cos\theta\sinh D_2 \\ \sin\theta\sinh D_1 & e^{C_2}\sin^2\theta & \sin\theta\cos\theta\sinh D_3 \\ \cos\theta\sinh D_2 & \sin\theta\cos\theta\sinh D_3 & e^{C_3}\cos^2\theta \end{pmatrix}$$

$$C_1(u,r,x^A) = \frac{C_{11}(u,x^A)}{r\sqrt{r}} + \frac{C_{12}(u,x^A)}{r^2} + \frac{C_{13}(u,x^A)}{r^2\sqrt{r}} + \frac{C_{14}(u,x^A)}{r^3} + O(r^{-7/2})$$

$$C_2(u,r,x^A) = \frac{C_{21}(u,x^A)}{r\sqrt{r}} + \frac{C_{22}(u,x^A)}{r^2} + \frac{C_{23}(u,x^A)}{r^2\sqrt{r}} + \frac{C_{24}(u,x^A)}{r^3} + O(r^{-7/2})$$

$$D_1(u,r,x^A) = \frac{D_{11}(u,x^A)}{r\sqrt{r}} + \frac{D_{12}(u,x^A)}{r^2} + \frac{D_{13}(u,x^A)}{r^2\sqrt{r}} + \frac{D_{14}(u,x^A)}{r^3} + O(r^{-7/2})$$

$$D_2(u,r,x^A) = \frac{D_{21}(u,x^A)}{r\sqrt{r}} + \frac{D_{22}(u,x^A)}{r^2} + \frac{D_{23}(u,x^A)}{r^2\sqrt{r}} + \frac{D_{24}(u,x^A)}{r^3} + O(r^{-7/2})$$

$$D_3(u,r,x^A) = \frac{D_{31}(u,x^A)}{r\sqrt{r}} + \frac{D_{32}(u,x^A)}{r^2} + \frac{D_{33}(u,x^A)}{r^2\sqrt{r}} + \frac{D_{34}(u,x^A)}{r^3} + O(r^{-7/2})$$

5次元重力波の無限遠での振る舞い

Solving Einstein equations

$$ds^2 = -\frac{Ve^B}{r^2}du^2 - 2e^B du dr + r^2 h_{AB}(dx^A + U^A du)(dx^B + U^B du)$$

Einstein 方程式

$$V = 1 + \frac{V_1(u,x^A)}{r\sqrt{r}} - \frac{m(u,x^A)}{r^2} + O(r^{-5/2})$$

$$B = \frac{B_1(u,x^A)}{r^3} + O(r^{-4})$$

$$U^A = \frac{U_1^A(u,x^A)}{r^2\sqrt{r}} + \frac{U_2^A(u,x^A)}{r^3} + \frac{U_3^A(u,x^A)}{r^3\sqrt{r}} + \frac{U_4^A(u,x^A)}{r^4} + O(r^{-9/2})$$

Solving Einstein equations 2

$$V = 1 + \frac{V_1(u, x^A)}{r\sqrt{r}} - \frac{m(u, x^A)}{r^2} + O(r^{-5/2})$$

trace part of $R_{AB} = 0$

$$\Rightarrow \quad V_1(u, x^A) = -\frac{2}{3}\left(\frac{1}{\sin\theta\cos\theta}\frac{\partial}{\partial\theta}(\sin\theta\cos\theta U_1^\theta) + \frac{\partial}{\partial\phi}U_1^\phi + \frac{\partial}{\partial\psi}U_1^\psi\right)$$

m はEinstein方程式からは決まらない。自由パラメータ。

Solving Einstein equations 3

$$U^A = \frac{U_1^A(u,x^A)}{r^2\sqrt{r}} + \frac{U_2^A(u,x^A)}{r^3} + \frac{U_3^A(u,x^A)}{r^3\sqrt{r}} + \frac{U_4^A(u,x^A)}{r^4} + O(r^{-9/2})$$

$R_{rA} = 0$ ➡

$$\begin{aligned}
U_1^\theta &= \frac{2}{5}\left[\frac{1}{\sin\theta\cos^2\theta}\frac{\partial}{\partial\theta}(\sin\theta\cos^2\theta C_{11}) + \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}D_{11} + \frac{1}{\cos\theta}\frac{\partial}{\partial\psi}D_{21} - \frac{1}{\sin\theta\cos\theta}C_{21}\right] \\
\sin^2\theta U_1^\phi &= \frac{2}{5}\left[\frac{1}{\sin\theta\cos\theta}\frac{\partial}{\partial\theta}(\sin^2\theta\cos\theta D_{11}) + \frac{\partial}{\partial\phi}C_{21} + \tan\theta\frac{\partial}{\partial\psi}D_{31}\right] \\
\cos^2\theta U_1^\psi &= \frac{2}{5}\left[\frac{1}{\sin\theta\cos\theta}\frac{\partial}{\partial\theta}(\sin\theta\cos^2\theta D_{21}) + \cot\theta\frac{\partial}{\partial\phi}D_{31} - \frac{\partial}{\partial\psi}(C_{11} + C_{21})\right],
\end{aligned}$$

等々。

U_4^A**はEinstein方程式からは決まらない。自由パラメータ。**

3. Asymptotic quantities

計量

$$ds^2 = -\frac{Ve^B}{r^2}du^2 - 2e^B dudr + r^2 h_{AB}(dx^A + U^A du)(dx^B + U^B du)$$

今の展開だと、

$$g_{uu} = -1 - \frac{V_1(u, x^A)}{r\sqrt{r}} + \frac{m(u, x^A)}{r^2} + O(r^{-5/2})$$

Bondi質量（係数は u = ー∞ でADM質量と一致するよう決定）

$$M_{\mathrm{Bond}}(u) = \frac{3}{16\pi}\int_{S^3} m(u, x^A)d\Omega$$

$$d\Omega = \sin\theta\cos\theta d\theta d\phi d\psi$$

Bondi mass loss law

$$R_{uu} = 0$$

$$\Rightarrow \quad \frac{d}{du}M_{\text{Bondi}} = -\frac{1}{16\pi}\int_{S^3}\left\{\left(\frac{\partial C_{11}}{\partial u}\right)^2 + \frac{\partial C_{11}}{\partial u}\frac{\partial C_{21}}{\partial u} + \left(\frac{\partial C_{21}}{\partial u}\right)^2 + \left(\frac{\partial D_{11}}{\partial u}\right)^2 + \left(\frac{\partial D_{21}}{\partial u}\right)^2 + \left(\frac{\partial D_{31}}{\partial u}\right)^2\right\} d\Omega$$

$$< 0$$

⟶ **Bondi質量は重力波により必ず減っていく。**

Bondi angular momentum

今の展開だと、

$$g_{t\phi} = \frac{1}{\sqrt{r}}\sin^2\theta U_1^\phi + \frac{1}{r}\sin^2\theta U_2^\phi + \frac{1}{r\sqrt{r}}\sin^2\theta U_3^\phi + \frac{1}{r^2}j^\phi + O(r^{-5/2})$$

$$g_{t\psi} = \frac{1}{\sqrt{r}}\sin^2\theta U_1^\psi + \frac{1}{r}\sin^2\theta U_2^\psi + \frac{1}{r\sqrt{r}}\sin^2\theta U_3^\psi + \frac{1}{r^2}j^\psi + O(r^{-5/2})$$

$$j^\phi = \sin\theta D_{11}U_1^\theta + \sin^2\theta C_{12}U_1^\phi + \sin\theta\cos\theta D_{13}U_3^\psi + \sin^2\theta U_4^\phi$$

$$j^\psi = \cos\theta D_{12}U_1^\theta + \sin\theta\cos\theta D_{13}U_1^\phi - \cos^2\theta(C_{11}+C_{12})U_3^\psi + \cos^2\theta U_4^\psi$$

Bondi角運動量（係数は u = ー∞ でADM角運動量と一致するよう決定）

$$J^\phi_{\rm Bondi}(u) = -\frac{1}{4\pi}\int_{S^3} j^\phi d\Omega$$

$$J^\psi_{\rm Bondi}(u) = -\frac{1}{4\pi}\int_{S^3} j^\psi d\Omega$$

Angular momentum loss by GWs

$$\frac{d}{du} J^{\phi}_{\mathrm{Bondi}}(u) = -\frac{1}{4\pi}\int_{S^3}\left[\left(\frac{\partial j^{\phi}}{\partial u}\right)_{\mathrm{radiation}} + \left(\frac{\partial j^{\phi}}{\partial u}\right)_{\mathrm{total\ derivative}}\right]$$

$$\begin{aligned}
\left(\frac{\partial j^{\phi}}{\partial u}\right)_{\mathrm{radiation}} = & -\frac{1}{4}\frac{\partial C_{11}}{\partial \phi}\frac{\partial C_{11}}{\partial u} - \frac{1}{8}\frac{\partial C_{12}}{\partial \phi}\frac{\partial C_{11}}{\partial u} - \frac{1}{8}\frac{\partial C_{11}}{\partial \phi}\frac{\partial C_{21}}{\partial u} - \frac{1}{4}\frac{\partial C_{21}}{\partial \phi}\frac{\partial C_{21}}{\partial u} - \frac{1}{4}\frac{\partial D_{21}}{\partial \phi}\frac{\partial D_{21}}{\partial u} \\
& + \frac{1}{10}\tan\theta\frac{\partial D_{31}}{\partial u}\left(\frac{\partial C_{11}}{\partial \psi} + \frac{\partial C_{21}}{\partial \psi}\right) + \frac{3}{20}\tan\theta\frac{\partial D_{31}}{\partial \psi}\frac{\partial C_{11}}{\partial u} + \frac{2}{5}\tan\theta\frac{\partial D_{31}}{\partial \psi}\frac{\partial C_{21}}{\partial u} \\
& - \frac{1}{2}\frac{\partial D_{11}}{\partial \phi}\frac{\partial D_{11}}{\partial u} + \frac{3}{20}\tan\theta\frac{\partial D_{11}}{\partial u}\frac{\partial D_{21}}{\partial \psi} - \frac{3}{20}\tan\theta\frac{\partial D_{11}}{\partial \psi}\frac{\partial D_{21}}{\partial u} - \frac{1}{4}\tan\theta\frac{\partial C_{11}}{\partial \psi}\frac{\partial D_{31}}{\partial u} \\
& - \frac{1}{10}\tan\theta\frac{\partial C_{21}}{\partial u}\frac{\partial D_{31}}{\partial \psi} - \frac{2}{5}\tan\theta\frac{\partial D_{11}}{\partial u}\frac{\partial C_{21}}{\partial \psi} - \frac{1}{4}\frac{\partial D_{31}}{\partial u}\frac{\partial D_{31}}{\partial \phi} + \frac{1}{4}\frac{\partial D_{31}}{\partial u}\frac{\partial D_{21}}{\partial \theta} \\
& + \frac{3}{20\cos^2\theta}\frac{\partial D_{11}}{\partial u}\frac{\partial}{\partial \theta}(\sin\theta\cos^2\theta D_{21}) - \frac{3}{20}\cos\theta\frac{\partial}{\partial \theta}(\tan\theta D_{31})\frac{\partial D_{21}}{\partial u} + \frac{1}{4}\frac{\partial D_{11}}{\partial \theta}\frac{\partial C_{21}}{\partial u} \\
& + \frac{3}{20\cos^2\theta}\frac{\partial D_{11}}{\partial u}\frac{\partial}{\partial \theta}(\sin\theta\cos^2\theta C_{11}) - \frac{3}{20}\frac{\partial C_{11}}{\partial u}\frac{\partial}{\partial \theta}(\tan\theta D_{11}) + \frac{3}{20\sin\theta\cos\theta}\frac{\partial C_{21}}{\partial u}\frac{\partial}{\partial \theta}(\sin^2\theta\cos\theta D_{11}) \\
& + \frac{3}{20}\sin\theta\frac{\partial C_{21}}{\partial \theta}\frac{\partial D_{11}}{\partial u} - \frac{1}{4}\frac{\partial D_{11}}{\partial \theta}\frac{\partial C_{21}}{\partial u} - \frac{1}{4}\sin\theta\frac{\partial D_{21}}{\partial \theta}\frac{\partial D_{31}}{\partial u} - \frac{3}{20\cos\theta}\frac{\partial}{\partial u}(C_{21}D_{11}),
\end{aligned}$$

全微分だからradiationには効かない

$$= -\frac{1}{4\pi}\int_{S^3}\left(\frac{\partial j^{\phi}}{\partial u}\right)_{\mathrm{radiation}} d\Omega$$

$J^{\psi}_{\mathrm{Bondi}}(u)$ についても同じ感じ

4. Asymptotic symmetry

漸近的対称性 ⇔ 座標条件、境界条件 を保つ 座標変換

$$\delta g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

Bondi 座標の定義（座標条件）

$$\delta g_{rr} = 0\,,\ \delta g_{rA} = 0\,,\ g^{AB}\delta g_{AB} = 0$$

$$\delta g_{uu} = O(r^{-3/2})\,,\ \delta g_{uA} = O(r^{-5/2})\,,\ \delta g_{AB} = O(r^{1/2})$$

重力波(outgoing wave condition)からくる境界条件

Poincare group

$$\delta g_{rr} = 0\,,\ \delta g_{rA} = 0\,,\ g^{AB}\delta g_{AB} = 0$$

translation

Lorentz group

$$\xi_r = f(u,x^A)e^B$$

$$\xi_B g^{AB} = f^A(u,x^A) - f(u,x^A)U^A + \int_r^\infty dr' e^B \frac{\partial f}{\partial x^B} g^{AB}$$

$$\xi_u = - - \frac{re^B}{3}\left(-\frac{\partial \xi_A}{\partial x^B} + \xi_C \Gamma^C_{AB} + \xi_r \Gamma^r_{AB}\right) g^{AB}$$

$$\delta g_{uu} = O(r^{-3/2})\,,\ \delta g_{uA} = O(r^{-5/2})\,,\ \delta g_{AB} = O(r^{1/2})$$

$$\frac{\partial f^A}{\partial u} = 0$$

$$\mathcal{D}_A f_B + \mathcal{D}_B f_A = -2\frac{\partial f}{\partial u} h^{(0)}_{AB}$$

$$\mathcal{D}_A \mathcal{D}_B f = \frac{1}{3}\mathcal{D}^2 f h^{(0)}_{AB}.$$

$$F(x^A) = \mathcal{D}_A f^A$$

$$f = -F(x^A)u/3 + \alpha(x^A)$$

$$\alpha(x^A) = a_u + a_x \sin\theta\cos\phi + a_y \sin\theta\sin\phi + a_z \cos\theta\cos\psi + a_w \cos\theta\sin\psi$$

Poincare covariance of Bondi angular momentum

translation

$$f = \alpha(x^A)\,,\ f^A = 0$$

$$j^\phi \to j^\phi - \alpha(x^A)\frac{\partial j^\phi}{\partial u} + (\delta j^\phi)_{\text{non radiation}} + (\text{total derivative terms})$$

$$-\alpha(x^A)\left(\frac{\partial j^\phi}{\partial u}\right)_{\text{radiation}} - \alpha(x^A)\left(\frac{\partial j^\phi}{\partial u}\right)_{\text{total derivative}}$$

$$-\alpha(x^A)\left(\frac{\partial j^\phi}{\partial u}\right)_{\text{total derivative}} + (\delta j^\phi)_{\text{non radiation}}$$

$$= -\frac{3}{4}m\frac{\partial \alpha}{\partial \psi} + (\text{supermomentum}) + (\text{total derivative})$$

translationに対しては 0

Poincare covariance of Bondi angular momentum 2

$$J^{\phi}_{\rm Bondi} \to J^{\phi}_{\rm Bondi} - \frac{1}{4\pi}\int_{S^3}\left[-\alpha(x^A)\frac{\partial j^{\phi}}{\partial u}\right]d\Omega + \frac{3}{16\pi}\int_{S^3} m(u,x^A)\frac{\partial \alpha}{\partial \phi}d\Omega$$

$$M_{ab} \to M_{ab} + 2P_{[a}\omega_{b]} - \left(f\frac{d}{du}M_{ab}\right)_{\rm radiation}$$

動的な時空でのPoincare変換

5. Summary

- 5次元 null infinity における漸近構造を明らかにした。

  1. 5次元時空で重力波でどのようにエネルギー、角運動量がどのように抜けていくか。
  2. 重力波込みの重力場が無限遠でどのように振舞うか。

- 定義した角運動量がPoincare群に付随する量であることを示した。

  1. 角運動量がwell-defined であることを示している。
  2. 重力波のもつ角運動量が無限遠で測れる。