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Oscillating Universe in Hořava-Lifshitz Gravity

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Motivation

◆Horava-Lifshitz gravity (HL gravity) P.Horava (2009) [1]

- $x^i \to bx^i, \mathrm{t} \to b^z t$: power-counting renormalizable ($z = 3$)
- higher curvature term in action ⟶ singularity avoidance

detailed balance condition
- restriction on action

⟶ impose (original)

⟶ relax (SVW)

◆Discussion in FLRW universe

✓ original : complete classification M. Minamitsuji (2009)[2]

✓ SVW : assumption of the e.o.s is not so clear

Analysis in SVW version without any no clear assumption

Preceding studies

◆ SVW version (with matter)

1. P. Wu and H. Yu (2009) [3]

   unstable static solution ⟶ expand, oscillation…

   Emergent universe : singularity avoidance

2. E. J. Son and W. Kim (2010) [4]

   inflation ⟶ second accelerated expansion

◆ Equation of state in HL gravity : e.o.s may be changed

$\rho_{rad} \propto a^{-4} \longrightarrow a^{-6}$: unknown running parameter

⟶ Analyze vacuum solution in SVW version

Horava-Lifshitz gravity

◆ Anisotropic scaling

$$x^i \to b x^i \qquad t \to b^z t$$
$$[x^i] = -1 \qquad [t] = -z$$

→

$$\mathcal{L}_{\rm int} = g\phi^n$$
$$[g] = 3 + z - \left(\frac{3-z}{2}\right) n$$

$z = 3$: power-counting renormalizable

◆ SVW : most general form of potential term for $z = 3$

T. P. Sotiriou, M. Visser and S. Weinfurtner (2009)[5]

$$\begin{aligned} \mathcal{V}_{\rm HL} \;=\; & 2\Lambda + g_1 \mathcal{R} + \kappa^2 \left(g_2 \mathcal{R}^2 + g_3 \mathcal{R}^i{}_j \mathcal{R}^j{}_i \right) + \kappa^3 g_4 \epsilon^{ijk} \mathcal{R}_{i\ell} \nabla_j \mathcal{R}^\ell{}_k \\ & + \kappa^4 \left(g_5 \mathcal{R}^3 + g_6 \mathcal{R}\, \mathcal{R}^i{}_j \mathcal{R}^j{}_i + g_7 \mathcal{R}^i{}_j \mathcal{R}^j{}_k \mathcal{R}^k{}_i + g_8 \mathcal{R} \Delta \mathcal{R} + g_9 \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk} \right), \end{aligned}$$

restriction on g_i : stability of flat background

$g_1 < 0,\ g_9 > 0,\ \lambda > 1$ ⟶ $g_1 = -1$ (time rescaling)

Application to FLRW universe

- background : homogeneous and isotropic

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right) ,$$

Hamiltonian constraint : $\kappa^2 = (1/M_{PL})^2 = 1$

$$H^2 + \frac{2}{(3\lambda-1)}\frac{K}{a^2} = \frac{2}{3(3\lambda-1)}\left[\Lambda + \frac{g_r}{a^4} + \frac{g_s}{a^6}\right]$$

$$g_r \equiv 6(g_3 + 3g_2)K^2 \ , g_s \equiv 12(9g_5 + 3g_6 + g_7)K^3$$

If $K \neq 0$, $g_s, g_r \neq 0$: discuss only a nonflat universe

- original : $g_s = 0, g_r < 0$
- SVW : $g_r,\ g_s$ are arbitrary

$g_r,\ g_s < 0$: corresponding to negative energy density

How to Classify

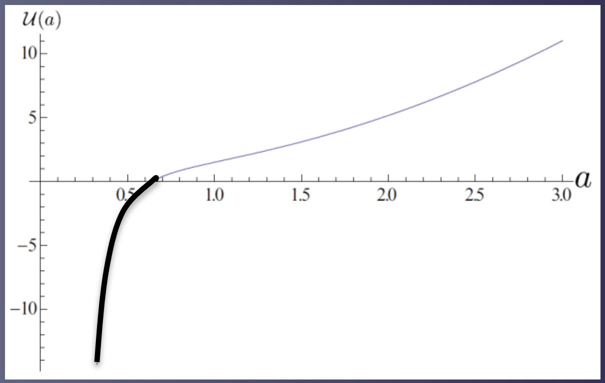
◆Hamiltonian constraint

$$\frac{1}{2}\dot{a}^2 + \mathcal{U}(a) = 0 \ , \ \mathcal{U}(a) = \frac{1}{3\lambda - 1}\left[K - \frac{\Lambda}{3}a^2 - \frac{g_{\rm r}}{3a^2} - \frac{g_{\rm s}}{3a^4}\right] .$$

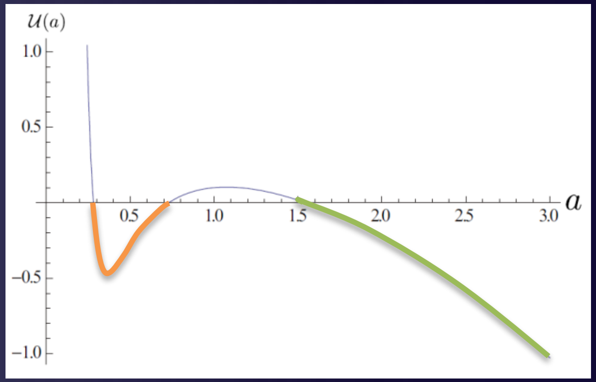
scale factor : particle with zero energy in $\mathcal{U}(a)$

→ possible range for scale factor : $\mathcal{U}(a) \leq 0$

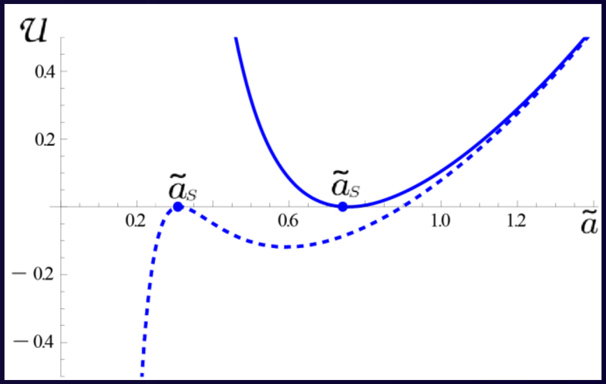
solutions



big bang → big crunch



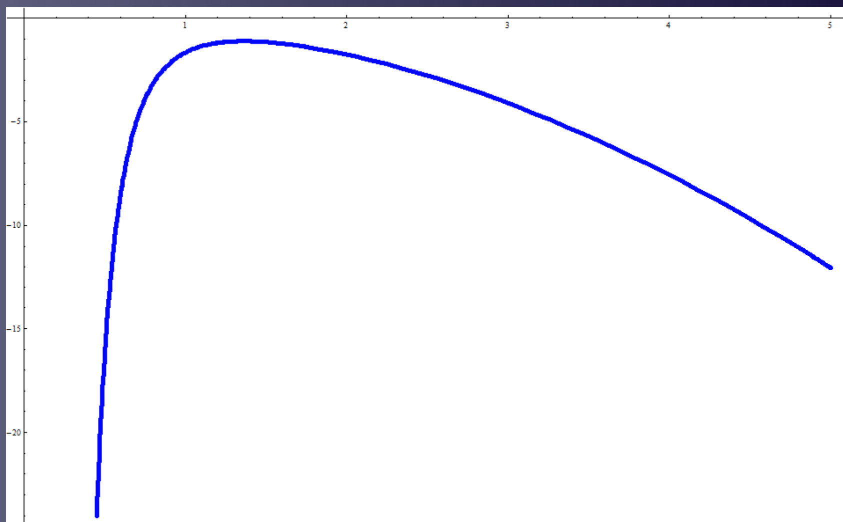
oscillation or bounce



stable static
unstable static

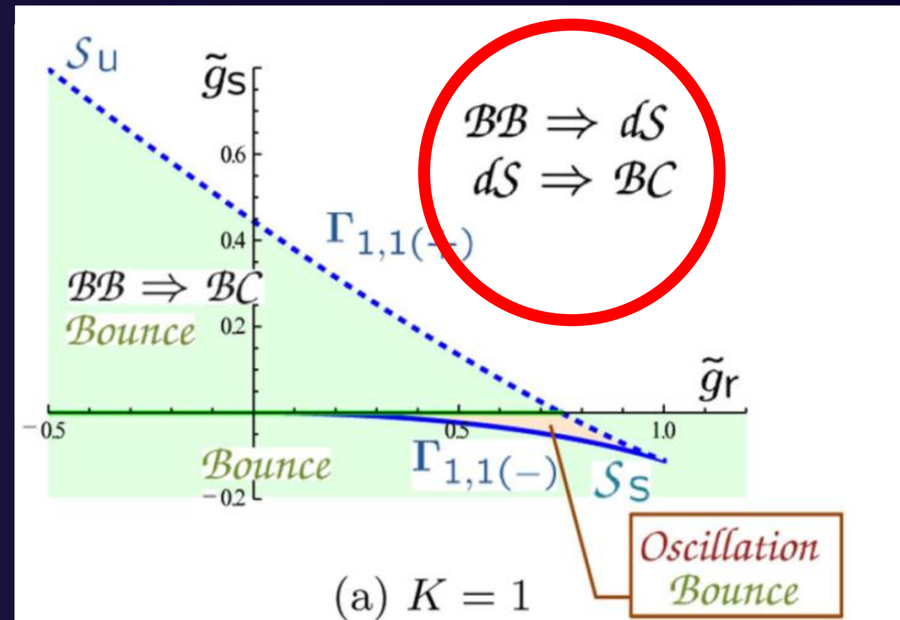
Classification ($\Lambda > 0, K = 1$)

1. Big bang → de Sitter



$\tilde{g}_s = 5, \tilde{g}_r = 5$

singularity is inevitable

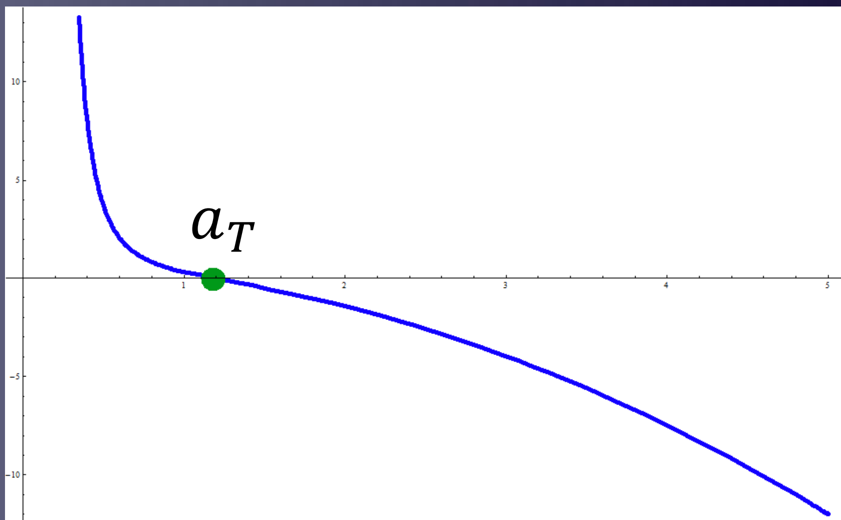


$\mathcal{BB}$: big bang $\mathcal{BC}$: big crunch
$d\mathcal{S}$: de Sitter $\mathcal{M}$: Milne
$\mathcal{S}_s$: stable static $\mathcal{S}_u$: unstable static

Big bang to de Sitter phase

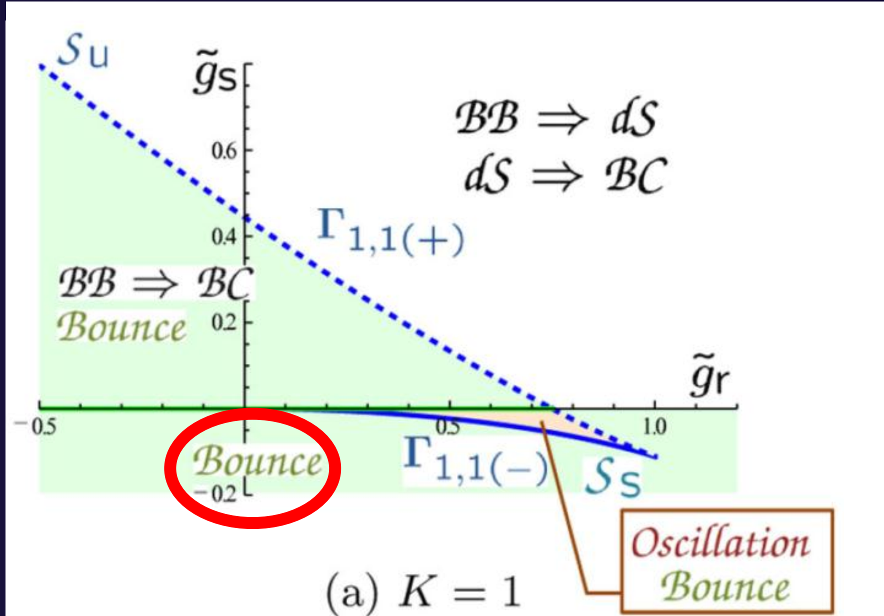
Classification ($\Lambda > 0, K = 1$)

2. bouncing universe



$\tilde{g}_s = -1, \tilde{g}_r = -1$

singularity avoidance



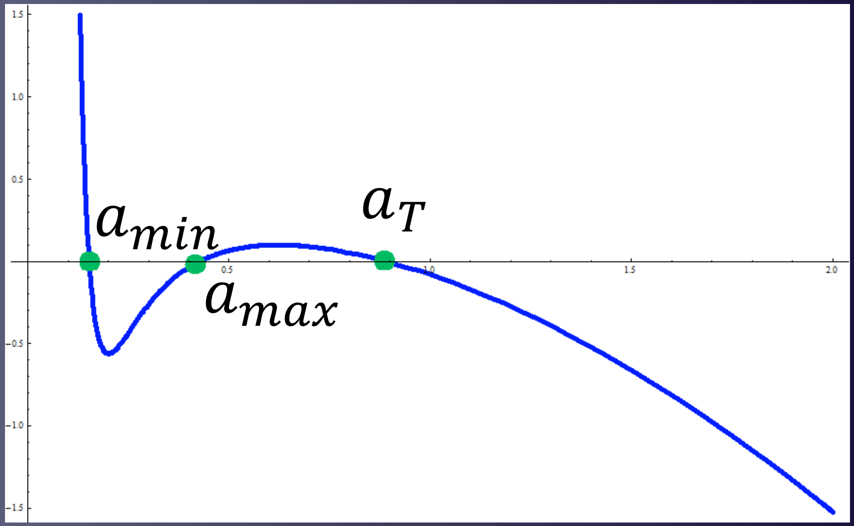
(a) $K = 1$

$\mathcal{BB}$: big bang $\mathcal{BC}$: big crunch
$d\mathcal{S}$: de Sitter $\mathcal{M}$: Milne
$\mathcal{S}_\mathrm{s}$: stable static $\mathcal{S}_\mathrm{u}$: unstable static

Solution with minimum value a_T : bouncing universe

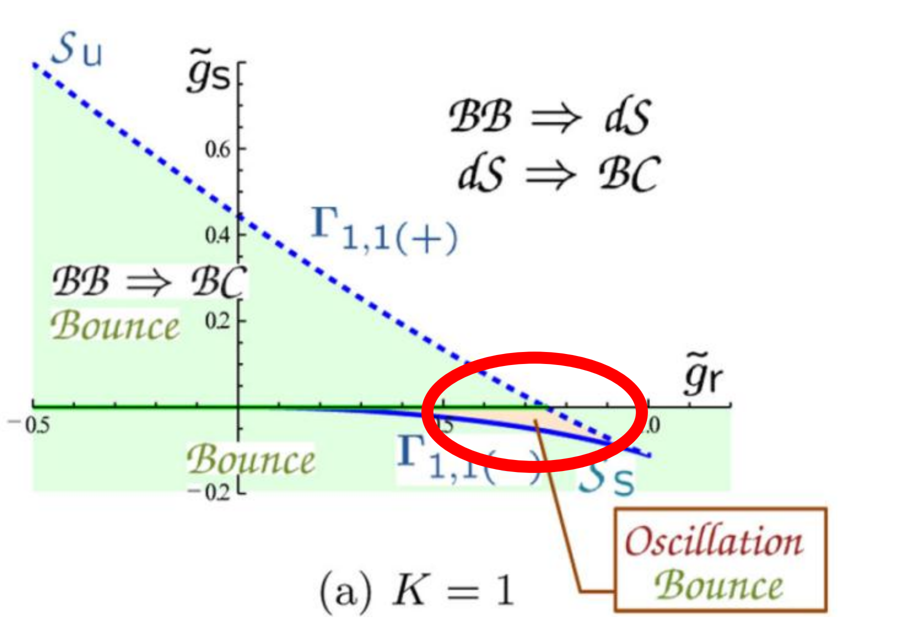
Classification ($\Lambda > 0, K = 1$)

3. oscillation and bounce



$\tilde{g}_s = -0.01,\ \tilde{g}_r = 0.5$

singularity avoidance



$\mathcal{BB}$: big bang $\mathcal{BC}$: big crunch

$d\mathcal{S}$: de Sitter $\mathcal{M}$: Milne

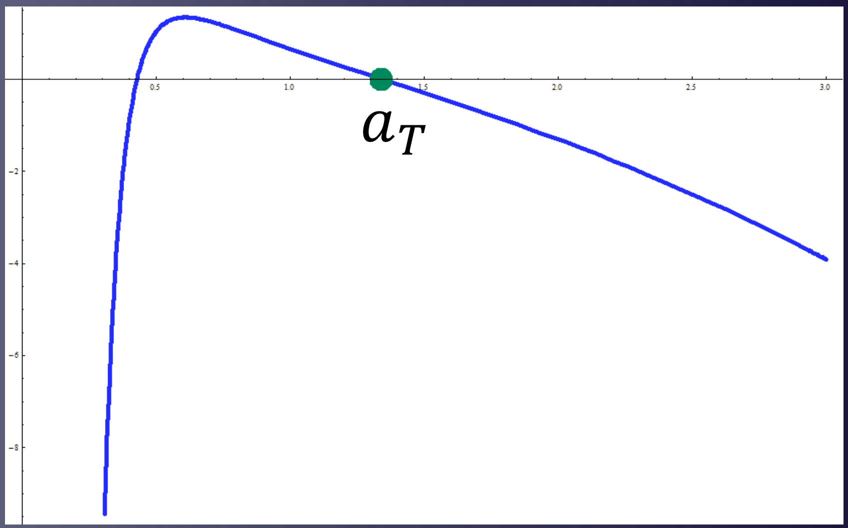
$\mathcal{S}_{\rm s}$: stable static $\mathcal{S}_{\rm u}$: unstable static

oscillating solution + bouncing solution

$g_s < 0$: sufficient condition for singularity avoidance

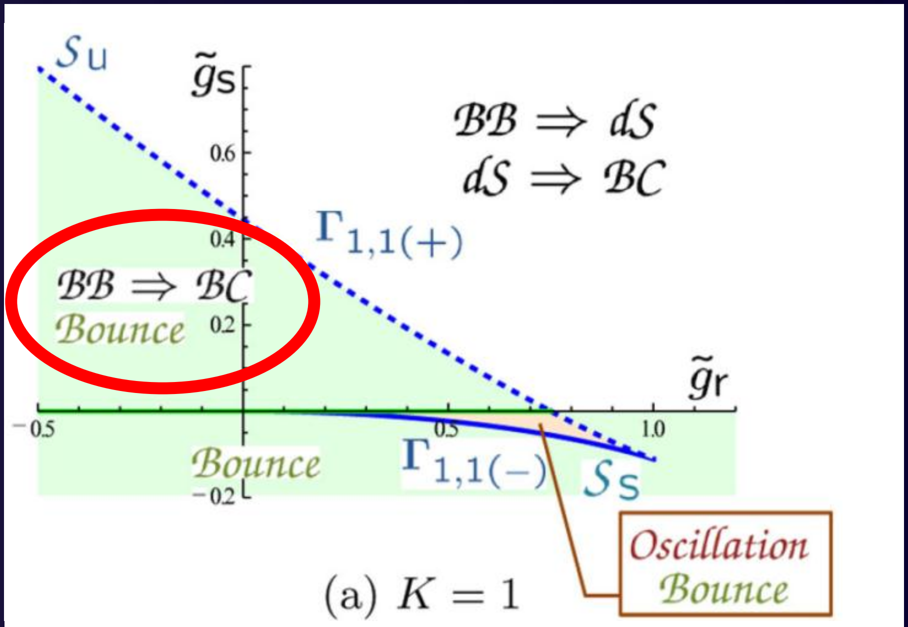
Classification ($\Lambda > 0, K = 1$)

4. bounce or big crunch



$\tilde{g}_s = -5,\ \tilde{g}_r = 1$

possibility of singularity avoidance



$\mathcal{BB}$: big bang $\quad$ $\mathcal{BC}$: big crunch

$d\mathcal{S}$: de Sitter $\quad$ $\mathcal{M}$: Milne

$\mathcal{S}_{\mathrm{S}}$: stable static $\quad$ $\mathcal{S}_{\mathrm{u}}$: unstable static

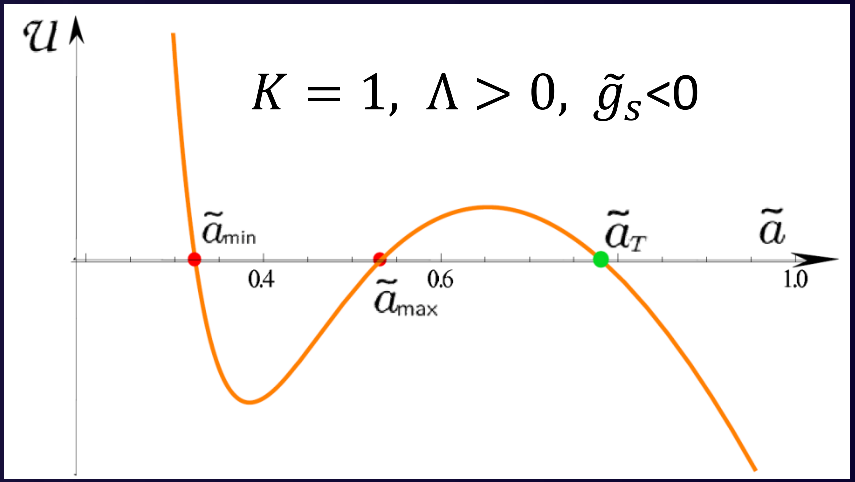
initial value $a \geq a_T$: bouncing universe

Toward a cyclic universe

◆ How to escape to macroscopic universe.

1. $K = 1, \Lambda > 0$: oscillation
2. quantum tunneling to a_T
3. de Sitter phase

⟶ macroscopic universe



◆ How to obtain cyclic universe.

1. Scalar field is responsible for inflation.
   - Before tunneling : behave as cosmological constant
   - After tunneling : slow-roll inflation ⟶ reheating ($\mathcal{U}(\infty)$>0)
2. upper bound and lower bound : cyclic universe

reheating — negative "stiff matter"

Summary and future works

- Classification vacuum FLRW universe in SVW HL gravity
  - Behavior of the universe is largely dependent on g_i

    sufficient condition for singularity avoidance : $g_s < 0$

    (oscillation, bounce)

- With matter : condition for singularity avoidance ?

$$g_s \to g_s + \underline{g_{stiff}}, \quad g_r \to g_r + \underline{g_{rad}}, \quad g_d \to \underline{g_{dust}}$$

energy density of matter (≥ 0)

- future work
  - discussion about stability of non-singular solution
    - Inhomogeneity
    - anisotropy : Bianchi type IX (in preparation)

reference

[1] P. Horava, Phys. Rev. D 79, 084008 (2009).

[2] M. Minamitsuji, Phys. Lett. B 684, 194 (2010).

[3] P. Wu and H. Yu, Phys. Rev. D 81, 103522 (2010).

[4] E. J. Son and W. Kim, arXiv:1003.3055.

[5] T. P. Sotiriou, M. Visser, and S. Weinfurtner, Phys. Rev.Lett. 102,251601 (2009)

[6] S. Carloni, E. Elizalde, and P. J. Silva, Classical Quantum Gravity 27, 045 004 (2010).

[7] Y. F. Cai and E. N. Saridakis, J. Cosmol. Astropart. Phys. 10 (2009) 020; G. Leon and E. N. Saridakis, J. Cosmol. Astropart. Phys. 11 (2009) 006.

detailed balance condition

$$\mathcal{V}_{\rm DB} = -\frac{3\kappa^2\mu^2\Lambda_W^2}{2(3\lambda-1)} + \frac{\kappa^2\mu^2\Lambda_W}{2(3\lambda-1)}\mathcal{R} - \frac{(4\lambda-1)\kappa^2\mu^2}{8(3\lambda-1)}\mathcal{R}^2 + \frac{\kappa^2\mu^2}{2}\mathcal{R}^{\,j}_{\,i}\mathcal{R}^{\,i}_{\,j} - \frac{2\kappa^2\mu}{\omega^2}\mathcal{C}^{ij}\mathcal{R}_{ij} + \frac{2\kappa^2}{\omega^4}\mathcal{C}_{ij}\mathcal{C}^{ij},$$

$$\mathcal{C}^{ij} := \epsilon^{ik\ell}\nabla_k\left(\mathcal{R}^{\,j}_{\ell} - \frac{1}{4}\mathcal{R}\delta^{j}_{\ \ell}\right), \quad \Lambda_W = -\frac{(3\lambda-1)}{(\mu^2\kappa^2)}$$

- Application to FLRW cosmology

$$g_{\rm r} = 6(g_3+3g_2) = -\frac{3\mu^2}{2(3\lambda-1)} < 0 \quad \text{for} \quad \lambda > 1/3$$

$$g_{\rm s} = 12(9g_5+3g_6+g_7)K = 0\,.$$

stability of flat background

- transverse traceless graviton

$$\omega^2_{\mathrm{TT}(\pm)} = -g_1 k^2 + g_3 \frac{k^4}{M^2_{\mathrm{PL}}} \pm g_4 \frac{k^5}{M^3_{\mathrm{PL}}} + g_9 \frac{k^6}{M^4_{\mathrm{PL}}}$$

  - stability In IR and UV : $g_1 < 0, g_9 > 0$

- longitudinal graviton

$$\left(\frac{3\lambda - 1}{\lambda - 1}\right) \omega^2_{\mathrm{L}} = g_1 k^2 + (8g_2 + 3g_3) \frac{k^4}{M^2_{\mathrm{PL}}} + (-8g_8 + 3g_9) \frac{k^6}{M^4_{\mathrm{PL}}}$$

  - ghost instability : $\frac{1}{3} < \lambda < 1$ ⟶ $\lambda > 1$
  - Instability in IR : not always pathological
    - ✓time scale
    - ✓non-projectable version

e.o.s of radiation

- generalized Maxwell action

$$S = \frac{1}{4}\int N\sqrt{g}\, dt\, d^3\vec{x}\left[\frac{2}{N^2}g^{ij}\left(F_{0i} - N^k F_{ki}\right)\left(F_{0j} - N^k F_{kj}\right) - G[B_i]\right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \ B_i = \frac{1}{2}\epsilon_{ijk} g^{kl} g^{km} F_{lm}$$

$$G[B_i] = a_1 B_i B^i + a_2 g^{ik} g^{jl} \nabla_i B_j \nabla_k B_l + a_3 g^{il} g^{jm} g^{kn} \nabla_i \nabla_j B_k \nabla_l \nabla_m B_n$$

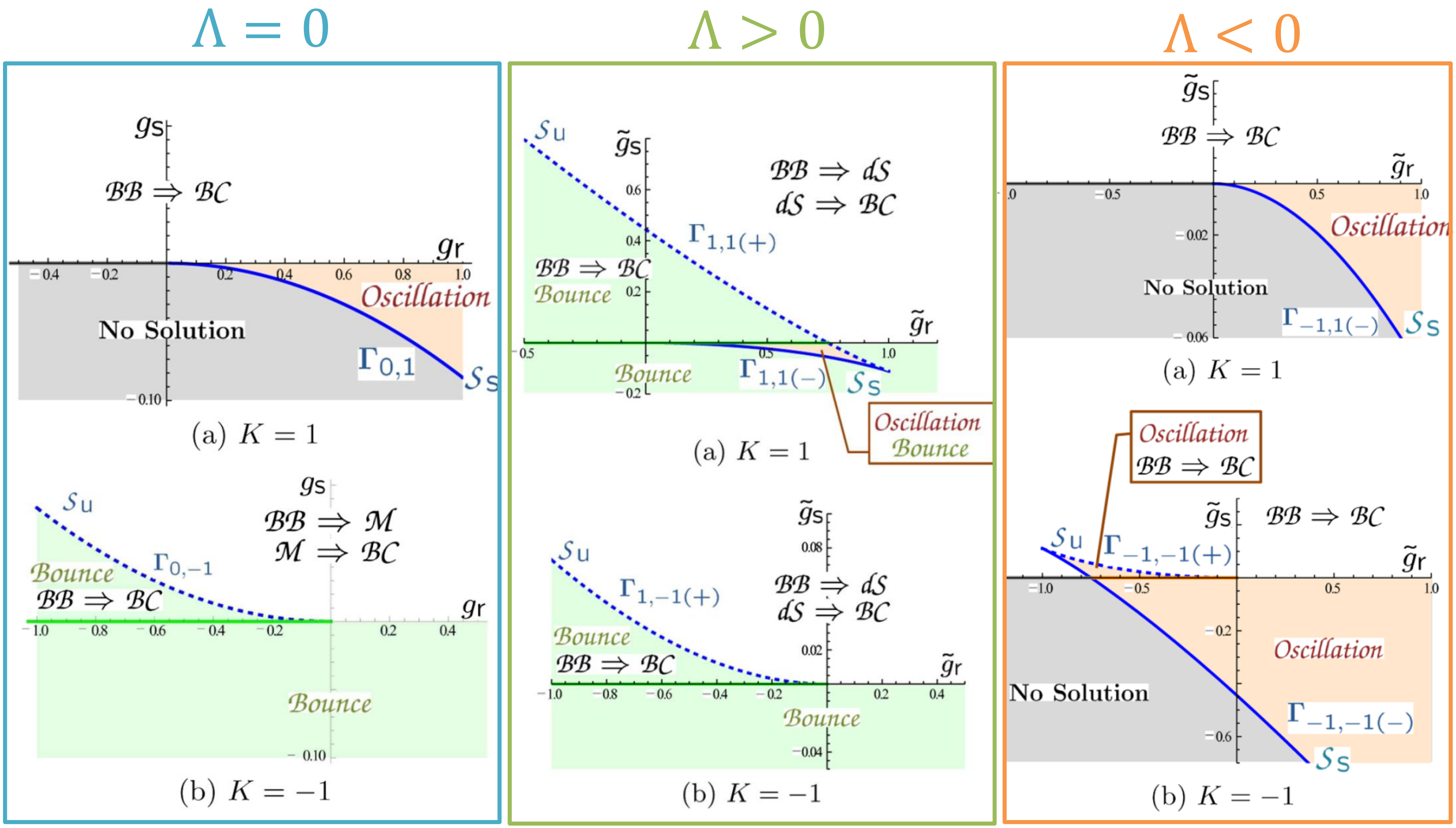
- dispersion relation for electromagnetic field

$$\omega^2 \simeq \frac{k^6}{M^4} + \alpha \frac{k^4}{M^2} + k^2 \quad \longrightarrow \quad \omega \sim \frac{k^z}{M^{z-1}} \ , k \sim a^{-1}$$

- depending on scale factor

$$\rho_{\rm rad} \sim \omega n \propto a^{-z-3}$$

n : number density ($\sim a^{-3}$)



$$\mathcal{U}(a) = \frac{1}{3\lambda - 1}\left[K - \epsilon\tilde{a}^2 - \frac{\tilde{g}_{\rm r}}{3\tilde{a}^2} - \frac{\tilde{g}_{\rm s}}{3\tilde{a}^4}\right]$$

If $\Lambda \neq 0$:

$$\frac{\Lambda}{3} = \frac{\epsilon}{\ell^2},\ \tilde{a} = \frac{a}{\ell},\ \tilde{g}_{\rm r} = \frac{g_{\rm r}}{\ell^2},\ \tilde{g}_{\rm s} = \frac{g_{\rm s}}{\ell^4}.$$